E504 APPLICABLE MATHEMATICS (YEAR 12) – 2006-2009

Rationale

Applicable Mathematics and other Year 12 TES subjects in Mathematics must fulfil a variety of needs. They should be both a satisfying culmination of the work of earlier years and an adequate preparation for tertiary studies. To do this, these subjects must present mathematics as an organised body of useful knowledge and provide students with the skills and confidence necessary to apply this knowledge in practical situations.

Applicable Mathematics meets these demands by offering studies in a range of topics in modern mathematics. These topics have the potential for useful application and are within the capabilities of the more mathematically able students of this age group. Students graduating from secondary school with a knowledge of these areas will appreciate the power of mathematics to provide a systematic way of understanding and interpreting the world around them.

Applicable Mathematics develops some of the more useful mathematical techniques to complement the calculus learned in Introductory Calculus.

One major theme is the solution of equations. This includes matrix methods for solving systems of linear equations, graphical methods for solving non-linear equations, and graphical linear programming.

The other theme is statistics and probability. In the statistics section students will learn to draw conclusions from statistical data using simple numerical and graphical techniques. The section on probability provides the framework for future studies of statistical inference.

Applicable Mathematics is therefore invaluable for those proceeding to tertiary studies in any of the courses for which the mathematical techniques described above are important.

Educational Objectives

This subject seeks to present mathematics as an organised body of knowledge which will provide students with a sound basis for later work in mathematics and other subjects. In addition the subject attempts to develop the following skills and attitudes.

Cognitive Objectives

Students will:

- recall mathematical facts and traditional terminology
- acquire mathematical concepts
- understand mathematical relationships
- acquire manipulative and computational skills
- use mathematical facts, traditional terminology, concepts, relationships and skills in routine ways
- comprehend information in oral and written forms including graphical, diagrammatic and tabular presentations
- select and use appropriate forms for representing mathematical data and relationships
- recognise and extend patterns and make conjectures, predictions and inferences from information given in oral and written forms
- understand and use deductive reasoning
- apply suitable mathematical techniques and problem solving strategies to both routine and non-routine situations
- select and use different technologies appropriately
- communicate mathematical ideas and results in both oral and written forms
- compare results with expectations and verify the suitability and reasonableness of a result.

Affective Objectives

It is highly desirable that students:

- develop an interest in mathematics and acquire a positive attitude towards its use and power
- show a willingness to participate and persevere in the learning of mathematics
- develop confidence in their ability to use mathematics effectively
- appreciate the benefits of using technology in mathematics
- display responsibility for their organisation, presentation and learning of mathematics
- interact in a constructive and cooperative manner with peers and teachers and respond constructively to advice.

Recommended Preparation

The recommended preparation for Applicable Mathematics is the successful completion of Introductory Calculus.

A set of *Counselling Notes* outlining the subject options in the Mathematics area for Year 11 and Year 12 students has been prepared and distributed to schools. Additional copies can be obtained from the Curriculum Council.

Teaching – Learning Program

The topics, or objectives within topics, can be taught in any order in keeping with the needs of teachers and students.

1. Systems of Linear Equations and Matrices (25 hours)

Gaussian elimination is studied as a systematic way of solving systems of linear equations in several variables. The coefficient matrix is introduced as a compact method of representing such a system, and row operations are studied as analogues of allowable equation manipulations.

Matrices are then studied in their own right and the basic properties of matrix algebra are examined. Matrix multiplication is illustrated by the study of linear transformations in the plane.

1.1 Model problems whose solutions reduce to systems of linear equations.

1.2 Solve systems of linear equations in two and three variables by elimination.

1.3 Examine the coefficient matrix of a system of linear equations, and the relationship between row operations on the coefficient matrix and rearrangements of the equations.

1.4 Solve systems of linear equations in two and three variables by reducing the coefficient matrix. Reduction to Echelon form is not always necessary.

1.5 Add, subtract and multiply matrices.

1.6 Examine the algebraic properties of matrix addition and multiplication.

1.7 Calculate the inverse of a 2 x 2 matrix and recognise a singular 2 x 2 matrix.

1.8 Given an n x n matrix, find the inverse and use it to solve the corresponding system of n linear equations in n unknowns.

1.9 Examine the geometric properties of 2 x 2 matrices as linear transformations in the plane.

Notes:

- Students are expected to be able to decide whether a system of linear equations has a unique solution, no solution, or infinitely many solutions. However, they are expected to find the solution only for the case where the solution is unique.
- Students should learn matrix arithmetic by practising on small matrices. However, it is necessary to reinforce their understanding of matrix operations by using appropriate technology. This will allow students to study the algebraic properties of matrix arithmetic, and to examine larger, more practical applications, without being overwhelmed by repetitive and tedious calculations.
- In studying matrix algebra, attention focuses on the properties of matrix multiplication, including non-commutativity.
- A knowledge of matrix inversion is essential for a proper understanding of systems of linear equations and linear transformations. However, it is not necessary for students at this level to study general inversion methods.
- Students should be able to construct, recognise and use transformation matrices for rotations, dilations, reflections and shears. They should be able to calculate the images of specific points. Images of general curves are not required. Matrix multiplication and matrix inversion should be related to composition and inversion of linear transformations. The relationship between the areas of the object and image should be considered.

2. Graphs and the Solution of Equations (18 hours)

Before graphs are used to solve equations, the transformational aspects of graphing should be studied. Graphs are used to find approximate solutions to equations for which algebraic methods may not be available.

Graphical techniques are also applied to simple linear programming problems.

Functions are restricted to those used in Introductory Calculus.

2.1 Investigate the effects of linear changes of variable on the graph of a function and on its algebraic formulation.

2.2 Solve problems of exponential growth and decay using log linear graphs and exponential regression.

2.3 Solve equations and inequalities graphically.

2.4 Model a variety of optimisation problems involving two variables as linear programs, and solve graphically.

Notes:

- Students should be encouraged to use graphics calculators and/or computer packages to explore the transformation ideas inherent in Section 2.
- Students are not required to know how to change the base of a logarithm. Problems should only involve base 10 or e, in which case the use of a calculator will suffice. Students are not expected to use log-linear graph paper.
- Students should compare the graphs of $y = f(x)$ and $Y = f(X)$, where f is a power, exponential or logarithm function, and $X = ax + b$ and $Y = cy + d$.
- Students will be expected to convert an exponential relationship: $y = Ae^{kx}$ to the linear: $\ln y = kx + \ln A$, and to draw and use the graph of $\ln y$ as a function of x.

3. Descriptive Statistics (20 hours)

In this section students will learn to draw conclusions from statistical data using simple numerical and graphical techniques. The relevance of these methods should be emphasised by using real data wherever possible.

3.1 Determine and use measures of central tendency (mean and median) for grouped and ungrouped data and the effect of changes of scale and origin and the effects of outliers on each. Calculate the proportional median for grouped data.

3.2 Determine and use measures of dispersion (range, interquartile range, variance and standard deviation) for grouped and ungrouped data. Examine the effects of changes of scale and origin and the effect of outliers.

3.3 Construct and interpret boxplots, noting their use in comparison of centres and spreads of data.

3.4 Define and use the correlation coefficient as a measure of linear association for bivariate data, and observe from examples that correlation does not imply causality.

3.5 Examine the invariance of the correlation coefficient under changes of origin and scale.

3.6 Find the least squares regression line, examine its properties and draw its graph.

3.7 Use the regression line for interpolation and extrapolation, and observe from examples the dangers of the latter.

3.8 Compare graphs of a fitted regression model and observed data to visually assess the appropriateness of the model.

3.9 Calculate and graph residuals for linear models and use them to assess the adequacy of regression models.

3.10 Use moving averages and residuals to detect and remove cycles in time series data. Use the least squares regression line and seasonal components for interpolation and extrapolation, where appropriate, for linear and periodic models.

3.11 Draw inferences from graphs of time series data.

Notes:

- Familiarity with most of the basic methods for presenting and summarising univariate data is assumed. However, it may be appropriate to review and consolidate such methods before proceeding with the new material here.
- For ungrouped data, the lower/upper quartile is the median of the scores to the left/right of the median.
- Some calculators (both graphic and scientific) give statistics for a population and a sample, using the 'n' and 'n-1' formulas respectively. Use of the sample values to estimate the population values is beyond the scope of the course. Students will be expected to use only the 'n' (population) option.
- For boxplots, the intent is for the median to be used as the centre and the upper and lower quartiles for the ends of the box. Students will not be required to draw a box plot for data grouped into intervals.
- Students should be able to standardise data. The correlation coefficient can then be introduced as the average of the products of the standard scores.
- The properties of the regression line are most clearly seen when standardised data is used. In this case, the regression line passes through the origin and its slope is the correlation coefficient.
- Properties of the regression line include the effect of changes in origin and scale on the equation of the line as in section 2.1.
- Students should also learn that the regression line minimises the sum of the squares of the residuals, but the derivation of this result is beyond the scope of this subject.
- Residuals are defined generally as the differences between observed and fitted values. This applies to both the linear model and the periodic model in 3.10. The idea of using residuals to check adequacy of models should be stressed. In the linear regression model the data points are random deviations from a line, and students should learn in an informal way how the actual spread of residuals can indicate where interpolation may be dangerous.
- Daily newspapers, financial periodicals and publications of the Australian Bureau of Statistics are good sources of data, particularly for time series.

4. Sets, Counting and Probability 18 hours)

Students will have been gradually exposed to the ideas of probability throughout their earlier years at school. In this section probability is given a more formal footing using set theory. This becomes the foundation for a study of random variables and their distributions. To emphasise the relevance of probabilistic methods, problems in this section should be realistic, and go beyond problems about coins and dice.

4.1 Use set notation, and the set operations of union, intersection, and complementation.

4.2 Use Venn diagrams to illustrate set concepts, and to solve problems in counting and probability.

4.3 Develop and use the addition and multiplication principles for counting, and display outcome sets in trees and systematic lists.

4.4 Investigate subsets of a set to develop and use the notation $\binom{n}{x}$ and ${}^nC_x$ for the number of subsets of size x in a set of n elements.

4.5 Develop the set-theoretic model of probability, with:

- outcomes as elements of a finite set, the sample space
- events as subsets
- intersection, union and complement of events
- probabilities assigned to events.

4.6 Develop the relationship

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

and use it to calculate probabilities.

4.7 Develop the concept of conditional probability and use the formula:

$$P(A \cap B) = P(A)\,P(B|A) = P(B)P(A|B)$$

to calculate probabilities.

4.8 Develop the concept of independent events and use the formulas:

$$P(A \cap B) = P(A)P(B);$$

$$P(B|A) = P(B); \quad P(A|B) = P(A).$$

Notes:

- In many cases the probability of an event is calculated by counting outcomes, and for this counting techniques are needed. However, long and convoluted counting problems are to be avoided. (Note: the use of permutation formulas is not in the syllabus).

- An important application of 4.4 is in establishing the formula for the probability function of a binomial random variable.
- Students should meet probabilistic arguments in a wide variety of circumstances by studying medical, social, physical, genetic, economic, industrial, engineering, computing, agricultural, political and gambling examples.
- Probabilities can be assigned by assuming that outcomes are equally likely, or by using data in the form of relative frequencies, or by using subjective opinions about such things as the chance of rain.
- When using set-theoretic methods to calculate probabilities, students should be encouraged to specify carefully the events of interest.
- Many of these ideas have been explored to some extent in lower school. The idea of formal mathematical modelling and the surprising nature of some answers should be emphasised.

  A good example is the calculation of the probability of NOT having a rare disease, given that a positive test result has been obtained. This can be high even when the test is accurate. Similar examples concern the probability of innocence given evidence suggesting guilt. Such problems can be solved using Venn diagrams, trees or tables.
- Concepts covered in section 4 are expected to be applied when using probabilities generated from distributions covered in section 5.

5. Random Variables and their Distributions (24 hours)

This section provides the basis for future studies at tertiary level of the formal processes of statistical inference. The fundamental concept of a random variable as a function is introduced, and probabilities associated with random variables are calculated.

5.1 Define discrete random variables as functions on a finite set of outcomes, and construct examples.

5.2 Determine the probability function of a discrete random variable, and use it to calculate probabilities.

5.3 Develop the concept of Bernoulli trials, and establish and use the binomial distribution for the number of successes.

5.4 Calculate and use probabilities associated with the Poisson random variable.

5.5 Develop the concepts of continuous random variables and their associated probability density functions.

5.6 Calculate and use probabilities associated with uniform, exponential and normal random variables.

5.7 Develop and use the normal approximation to the binomial and the Poisson distributions.

Notes:

- The shorthand notation $P(X=x)$ should be introduced and its meaning explained. The probability distribution for a discrete random variable should be described as a table listing each value x in the range of the random variable and the corresponding probability $P(X = x)$.
- Students have experience with sampling from a population, and this can be used to define interesting random variables. For example, in an investigation into the number and distribution of computers in the homes of students at a school of size 1000, the outcome set could be the set of all samples of size 30, and the random variable could give the number of students in such a sample who have a computer at home.
- Examples of Bernoulli trials include sampling with replacement for a binary characteristic. This approximates opinion polling. Other examples include the presence or absence of genes, guessing in multiple choice exams, and the incidence of defective items in production lines.
- The formula for the probability function for the binomial variable (5.3) should be linked to the expansion of $(p+q)^n$ by the binomial theorem.
- The Poisson random variable is a probability model for the distribution of the number of occurrences of a randomly occurring event in a fixed interval of time or unit of space. Plausible examples are the number of radioactive emissions per unit time, the number of equipment failures per unit time, the number of snags in yarn per unit length or the number of weeds in a unit area. The Poisson can also be used to approximate the binomial when n is large and p is very small.
- Plausible examples of normally distributed random variables include height, weight and intelligence measures. Plausible examples of uniformly distributed random variables include random numbers and differences between scheduled and actual arrival times for buses. Plausible examples of exponentially distributed random variables include times between radioactive emissions, waiting times for equipment failures, interarrival times at queues of various kinds, and distances between snags in yarn.
- Computers can well illustrate the quality of the normal approximation to the binomial and Poisson distributions. When making such an approximation, the correction for continuity is required.
- Concepts covered in section 4 are expected to be applied when using probabilities generated from distributions covered in section 5.

Time Allocation

The subject has been designed to be completed through a structured education program of approximately 110 hours in any suitable contexts and series of learning experiences. Typically the subject will be studied over the period of one school year. For administrative reasons schools wishing to vary this delivery pattern (e.g. over a shorter period or over a longer period up to two school years) are required to notify the Chief Executive Officer of the Curriculum Council.

Subject Completion

Students must complete the school's structured educational and assessment program for a subject in order to be eligible to receive a grade unless there are exceptional and justifiable circumstances. In situations where the school considers that insufficient information has been gathered to justify the award of a grade for the subject, a result of U (for unfinished) should be allocated. The Curriculum Council offers the flexibility for the U to be converted to a grade after the final grades have been submitted. Further details on assessment and grading are provided in Volume I of the Syllabus Manuals.

Examination Details

The examination will consist of one written paper of three hours duration.

The written paper will consist of a mixture of short routine questions and more challenging non-routine questions reflecting the syllabus content weightings in Table 1 of the Assessment Structure. There will be no choice of questions.

Resources:

- Notes on two unfolded A4 sheets of paper
- Curriculum Council *Mathematical Formulae and Statistical Tables* book
- Drawing instruments and templates

Calculators satisfying the conditions set by the Curriculum Council for this subject, which are listed on the Curriculum Council website: http://www.curriculum.wa.edu.au/files/doc/139775_2.doc

Note: Personal copies of the tables book should not contain any handwritten or typewritten notes, symbols, signs, formulas or any other marks (including underlining and highlighting), except the name and address of a candidate, and may be inspected during the examination.

Assessment Structure

Assessment structures are an integral part of all Accredited Subjects.

The structure specifies:

1. the components and learning outcomes to be included in assessment
2. weightings to be applied to these components
3. the types of assessment considered appropriate for the subject.

Table 1

Syllabus Content	**Weighting percentage**
Systems of linear equations and matrices	20-30
Graphics and the solution of equations	15-20
Descriptive statistics	15-20
Sets, counting and probability	15-20
Random variables and their distributions	20-25

Table 2

Learning Outcomes	**Weighting percentage** (relative significance of learning outcomes)
Lower order cognitive objectives *	45-55
Higher order cognitive objectives **	45-55

* which include recall of skills, acquisition of concepts and routine use of mathematical knowledge.

** which describe processes such as generalisation, justification, deduction and the application of mathematical techniques and problem solving strategies in non-routine ways.

Table 3

Types of Assessment	**Weighting percentage**
Extended pieces of work	15-30
Other forms of assessment	0-20
Tests and Examinations	50-75

The assessment program must provide students with the opportunity to demonstrate achievement of the requirements of the subject.

AND

Students must complete the requirements of the subject.

Supporting Notes

1. In the assessment program developed by schools, due attention must be given to the full range of educational objectives. The Syllabus Committee recommends that, in order to assess the achievement of the higher order cognitive objectives, **at least three (3)** extended pieces of work be included in the assessment program.
2. In many assessment tasks it is appropriate and desirable for students to work cooperatively and obtain help from various sources. However, assessment in mathematics should focus on the level of knowledge and understanding a student has attained and not the manner in which the learning has taken place. In all cases where students can collaborate or obtain outside help for assessable work, schools must ensure that marks awarded are measures of the student's own knowledge and understanding. Accordingly, validation is regarded as an integral part of the assessment procedure.
3. Student performance in the subjects will be graded on a scale of A, B, C, D and E according to the degree to which students have achieved the stated objectives of the subject.

   A set of grade-related descriptors, which indicate the standard of student achievement required for each grade, is available for these subjects and should be used when grading student performance.

Notes on Table 3

Extended Pieces of Work: These are assessment tasks which allow students the opportunity to demonstrate higher order cognitive skills such as verification, justification, generalisation, deduction, interpretation and application. Only tasks which meet these requirements can be included as EPWs. They may involve an in-class and/or out of class component and may be completed over an extended period of time. The intent is to set tasks that can be completed free from the pressures of time. Careful consideration of the time allocated to complete such tasks is therefore essential. Projects which involve higher order skills may be considered as EPWs.

Other Forms of Assessment: There are other forms of assessment that may be used to determine the extent to which students have achieved the objectives of the course. Assignments, projects, checklists, homework, teacher observation, and oral presentations should be included in this category.

Notes:

- While out of class assessment tasks at times allow (in fact may require) students to utilise resources beyond those typically available while being assessed under supervised conditions, it is expected that work finally submitted for assessment should be both known to and understood by the students concerned.
- Schools are responsible for determining procedures which should be applied to eliminate the likelihood of cheating, plagiarism, collusion and the like.
- School-based initiatives to exercise some form of ‘control’ over out-of-class assessment tasks are considered desirable as a means of both protecting students and retaining the integrity of using school assessments for external purposes.
- The following are example of ‘controls’ that may be legitimately used:
  - professional judgement
  - in-class supervised tasks with open access to notes and other resources
  - achievement on ‘at home’ tasks measured by a brief in-class test on key concepts
  - research done out of class with reports completed in class under supervised conditions
  - teacher/student interview
  - a combination of in-class and out-of-class components.

Grade-Related Descriptors

Grade-Related Descriptors describe the student performance standards that are used to award grades in this subject. Schools delivering this subject have been provided with a copy of the document. Additional copies may be purchased from the Curriculum Council.