

Grade descriptors and grading package

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Grade descriptors and grading

Mathematics

Unit 3A and 3B

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The descriptors need to be read with the information and sample tasks that follow.

A	Behaviours associated with Grade A, representing excellent achievement.	
	General behaviour	Specific form of the behaviour (examples)
Identifies and organises relevant information.	Identifies and organises relevant information from dense and scattered information:	identifies and states the equations or in-equations that adequately represent quantities that vary in a particular problem.
	Identifies and organises relevant information from key elements in ambiguous data:	identifies a cycle in data that does not follow the usual pattern.
Chooses effective models and methods and carries through the methods correctly.	Carries through extended responses:	correctly connects the many stages when dealing with time series data in both tabular and graphical form.
	Carries through deductive reasoning:	describes the effect on the linear model of removing a single data point from a residual plot.
	Applies mathematics in unpractised ways:	determines the cycle in a time-series graph by counting points, when the cycle is not obvious.
	Translates between representations in unpractised ways:	determines the seasonal adjustment values from a graph rather than a table.
Obeys mathematical conventions and attends to accuracy.	Uses inequality signs correctly:	attends to open or closed intervals.
	Rounds to suit contexts and specified accuracies in extended responses:	rounds to the nearest 50 thousand.
Links mathematical results to data and contexts to reach reasonable conclusions.	Recognises specified conditions in extended responses:	defines intervals of the domain for which certain conditions apply, such as $k(x) > c(x)$ for $1.0051 < x \leq 4$
	Determines limitations of a model:	excludes a section of data before fitting the rest of the data to a function.
Communicates mathematical reasoning, results and conclusions.	Shows main steps in reasoning:	graphs two functions, finds their points of intersection and defines the solution set for the required conditions.

The descriptors need to be read with the information and sample tasks that follow.

B	Behaviours associated with Grade B, representing high achievement.	
	General Behaviour	Specific form of the behaviour (examples)
Identifies and organises relevant information.	Identifies and organises relevant information from dense and scattered information:	identifies all relevant information such as a graph representing time series data.
Chooses effective models and methods and carries through the methods correctly.	Solves unstructured problems:	solves simultaneous equations graphically and applies boundary conditions.
	Carries through deductive reasoning:	uses graphical method to solve simultaneous equations; makes appropriate conjectures concerning trend lines.
	Determines the effects of changed conditions:	applies boundary conditions according to word problem.
	Combines interpretation of multiple representations:	compares the table, graph and regression equation to make a conjecture concerning the slope or trend.
Obeys mathematical conventions and attends to accuracy.	Rounds to suit contexts and specified accuracies in extended responses:	rounds to 4 decimal places.
Links mathematical results to data and contexts to reach reasonable conclusions.	Interprets multi-dimension quantities:	interprets a gradient of a 'fertility rate' versus 'time' graph.
	Determines limitations of a model:	mentions points that differ greatly from the graph of the fit when fitting the data of a scatter graph to a function.
Communicates mathematical reasoning, results and conclusions.	Justifies by stating properties that have been applied:	explains using the pattern on a graph why a 12-point average was used to determine a trend line.

The descriptors need to be read with the information and sample tasks that follow.

C	Behaviours associated with Grade C, representing satisfactory achievement.	
	General Behaviour	Specific form of the behaviour (examples)
Identifies and organises relevant information.	Identifies and organises relevant information from information that is grouped together and is narrow in scope:	uses the residual plot of a given model to determine the suitability of the model to fit a set of data.
	Interprets single relative terms:	picks the most likely point to have been plotted in the wrong place using a residual plot.
Chooses effective models and methods and carries through the methods correctly.	Answers structured questions that require short responses:	sketches a given trend line on the same axes as a time-series plot.
	Calculates specific cases of generalisations:	uses a regression line to predict a single value for the dependent variable.
	Makes commonsense connections:	describes what the patterns of set of data mean.
	Applies mathematical methods in practised ways:	calculates the moving average; uses a calculator to produce a mean or standard deviation from a set of data; uses the midpoints of intervals in a grouped data set to calculate a mean or standard deviation.
	Interprets mathematics in practised ways:	uses a calculator to calculate a regression line from a given data set.
Obeys mathematical conventions and attends to accuracy.	Applies basic conventions for diagrams, graphs and maps:	labels axes and significant points on a sketch graph.
	Defines introduced variables:	rewrites a linear regression line $y = ax + b$ in terms of the two variables on the scatter graph, e.g. the independent variable (year number) and dependent variable (fertility rate) $Fertility\ rate = 0.042 \times year\ number + 0.9773$
	Rounds to suit contexts and specified accuracies in short responses:	gives money answers (\$) to two decimal places without being directed.
	Attends to accuracy:	calculates moving averages using a calculator.
Links mathematical results to data and contexts to reach reasonable conclusions.	Recognises specified conditions in short responses:	gives the average to the nearest unit since the data set was recorded to the nearest unit.
	Attends to units in short responses:	refers to units such as 'minutes' when giving the answer for a mean or standard deviation.
Communicates mathematical reasoning, results and conclusions.	Shows working, including expressions entered into a calculator:	uses a regression line to predict a value; calculates a moving average.
	Justifies with a simple or routine statement:	states ' <i>the residuals are more random</i> '.
	Uses basic symbols:	includes appropriate notation as part of the answer such as $\leq, \bar{x}, \$$

The descriptors need to be read with the information and sample tasks that follow.

D	Behaviours associated with Grade D, representing limited achievement.	
	General Behaviour	Specific form of the behaviour (examples)
Identifies and organises relevant information.	Identifies and organises relevant information from information that is grouped together and is narrow in scope:	describes the trend of a graph in terms such as ' <i>is increasing</i> ' or ' <i>is decreasing</i> '.
	Does not recognise that a question has more than one answer:	misses other solutions despite the use of a plural noun in a question.
Chooses effective models and methods and carries through the methods correctly.	Carries through a single thread of reasoning:	decides whether a set of residual plots are randomly placed relative to the zero line.
	Translates between representations in practised ways:	selects the correct linear equation (algebraic) to match a given trend line (graphical).
	Interprets in practised ways:	describes a pattern of points as increasing or decreasing.
Obeys mathematical conventions and attends to accuracy.	Correctly enters data into a calculator:	enters data into single variable data set to calculate statistics such as mean and standard deviation.
Links mathematical results to data and contexts to reach reasonable conclusions.	Attends to units in short responses:	refers to units such as minutes when giving the answer for a mean or standard deviation.
Communicates mathematical reasoning, results and conclusions.	Shows working, including expressions entered into a calculator:	uses a regression line to predict a value; calculates a moving average.
	Uses basic symbols:	includes appropriate notation as part of the answer such as \leq , \bar{x} , \$

The descriptors need to be read with the information and sample tasks that follow.

E	Behaviours associated with Grade E, representing limited achievement.	
	General Behaviour	Specific form of the behaviour (examples)
Identifies and organises relevant information.	Identifies and organises relevant information from information that is grouped together and is narrow in scope:	describes the trend of a graph in terms such as ' <i>is increasing</i> ' or ' <i>is decreasing</i> ' with limited success.
	Does not recognise that a question has more than one answer:	misses other solutions despite the use of a plural noun in a question.
Chooses effective models and methods and carries through the methods correctly.	Carries through a single thread of reasoning:	decides whether a set of residual plots are randomly placed relative to the zero line.
	Translates between representations in practised ways:	selects the correct linear equation (algebraic) to match a given trend line (graphical).
	Interprets in practised ways:	describes a pattern of points as ' <i>increasing</i> ' or ' <i>decreasing</i> '.
Obeys mathematical conventions and attends to accuracy.	Enters data correctly into a calculator:	uses single variable data to calculate statistics such as mean and standard deviation inaccurately.
Links mathematical results to data and contexts to reach reasonable conclusions.	Attends to units in short responses:	refers to the units such as minutes when giving the answer for a mean or standard deviation.
Communicates mathematical reasoning, results and conclusions.	Shows working, including expressions entered into a calculator:	does not use a regression line to predict a value; calculates a moving average inaccurately.
	Uses basic symbols:	does not include appropriate notation such as \leq , \bar{x} , \$

Grading student achievement in Mathematics

Grades will be based on students' performance on response and investigation questions which are referred to as '*tasks*' in this document.

A and B grades will be based on evidence of higher-level mathematical behaviours, as indicated in the grade descriptors. Tasks that allow for such behaviours are labelled '*complex*' in this document. C, D and E grades will be based on the extent to which students show lower-level mathematical behaviours, as indicated in the grade descriptors. Tasks that allow for these behaviours are labelled '*simple*' in this document.

The assessment and grading process in Mathematics

The following steps may assist teachers in assessing student achievement in Mathematics and grading students at the end of a unit.

Step 1: Design assessments

Design assessments so they contain simple and complex tasks as described above. Make quick judgements only as to whether tasks are simple or complex. Some tasks may be left undecided. Teachers can refer to the examples of simple and complex tasks given in Table 2 (page 12). Sample test questions are provided in this document (see pages 14 – 33).

Step 2: Prepare the marking keys

Prepare marking keys that reward the assessable behaviours that are embedded in the grade descriptors, that is, assign marks for:

- identifying and organising relevant information
- choosing effective models and methods and carrying through methods correctly
- obeying conventions and attending to accuracy
- linking mathematical results to data and contexts to reach reasonable conclusions
- communicating mathematical reasoning, results and conclusions.

Step 3: Administer and mark the assessments

Use follow-through marking, where all correct behaviours are rewarded and errors are penalised once only.

Step 4: Review the assessments

Review the classification of tasks as *simple* or *complex* after marking. Note reasons for complexity as these could be used to decide cut-off grades (Step 7) and/or to inform teaching.

Step 5: Record students' marks

Record students' total marks for all assessments.

Step 6: Rank students

At the end of a unit, rank students on the basis of their total scores for all assessments. Use professional judgement to choose provisional cut-offs for each grade.

Step 7: Decide the grade cut-offs

Inspect scripts of students near the provisional cut-offs. Look at students' performance on 4 or 5 tasks that effectively discriminated between students. Judgements are based on the complex and not on the simpler tasks, where an assessment item has both types. Adjust final cut-offs using the grade descriptors, and Table 1 on page 10. The lowest-ranked student who satisfies requirements for an A grade, and all students ranked above, are awarded an A. The same pattern applies to the other grades.

Table 1: A rubric to assist in allocating grades

A	B	C	D	E
Students demonstrate appropriate mathematical behaviour on $\geq 50\%$ of complex tasks. Appropriate mathematical behaviour on simple tasks is assumed.	Students demonstrate appropriate mathematical behaviour on $\geq 20\%$ complex tasks and on $\geq 80\%$ of simple tasks.	Students demonstrate appropriate mathematical behaviour on $\geq \frac{2}{3}$ (67%) of simple tasks.	Students demonstrate appropriate mathematical behaviour on $\geq 50\%$ of simple tasks.	Students demonstrate appropriate mathematical behaviour on $< 50\%$ of simple tasks.

Figure 1 on page 11 summarises the grading process. Consensus and moderation processes will be used to assist comparability between schools.

Summary of the recommended grading process

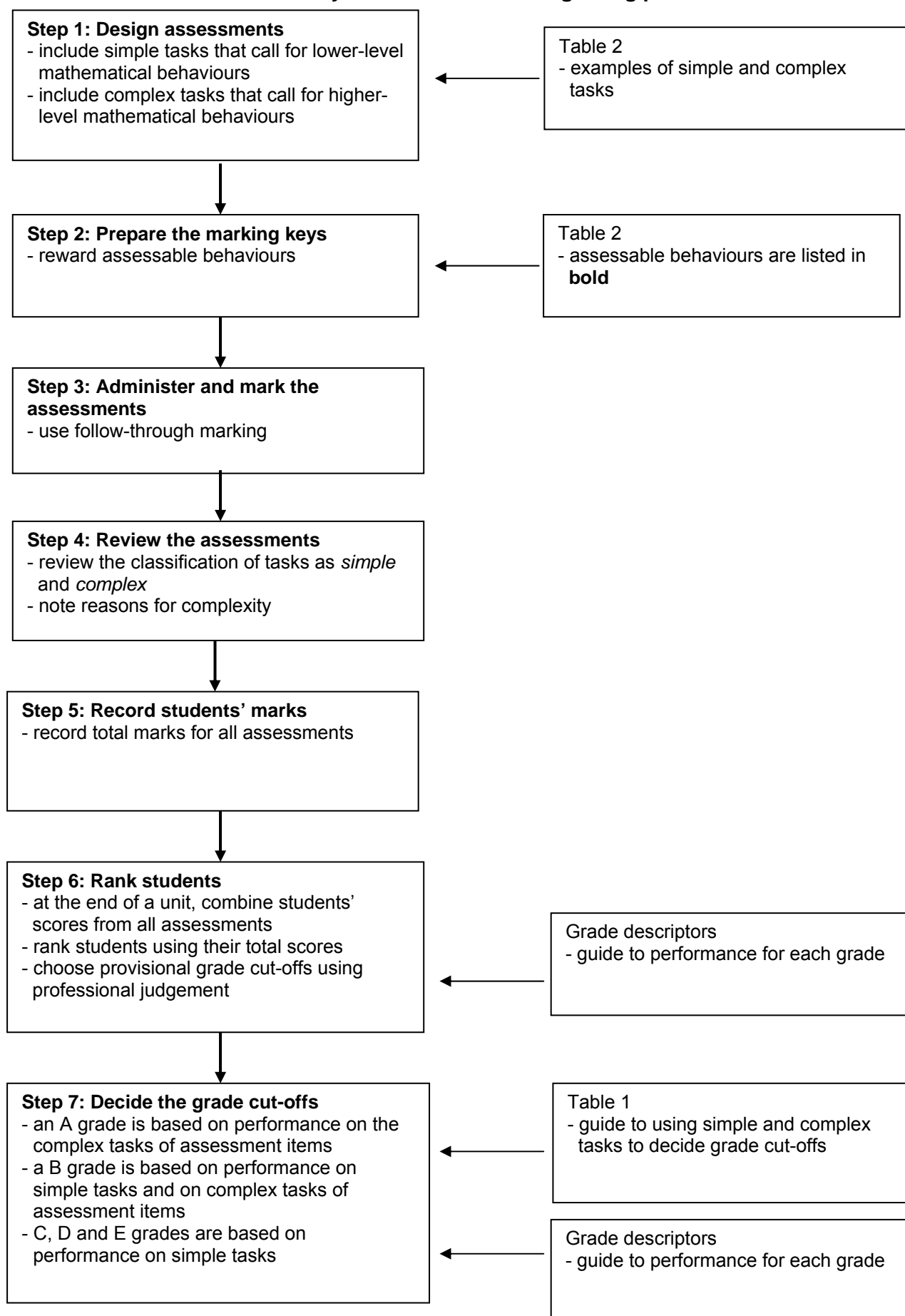


Figure 1: Summary of the recommended grading process

Table 2: Examples of simple and complex tasks 3AMAT and 3BMAT

Simple tasks provide evidence of these behaviours:	2005 Applicable TEE questions	Complex tasks provide evidence of these behaviours:	2005 Applicable TEE questions
identifies and organises relevant information		identifies and organises relevant information	
<ul style="list-style-type: none"> identifies relevant information in information that is narrow in scope 	4a, 11a	<ul style="list-style-type: none"> identifies relevant information from dense information 	3, 14e
<ul style="list-style-type: none"> interprets single relative terms (lowest, more than etc.) 	4c	<ul style="list-style-type: none"> interprets multiple relative terms (lowest, more than etc.) 	11biii
		<ul style="list-style-type: none"> identifies key elements in ambiguous* data 	14a
<ul style="list-style-type: none"> recognises a question has more than one answer from the plural noun 	13a		
chooses effective models and methods and carries through methods correctly		chooses effective models and methods and carries through methods correctly	
<ul style="list-style-type: none"> answers structured questions that require short responses 	14d	<ul style="list-style-type: none"> solves unstructured problems using sub-problems 	3
<ul style="list-style-type: none"> carries through a single thread of reasoning 	4a, 14b	<ul style="list-style-type: none"> carries through deductive reasoning 	3, 4c, 13bi
<ul style="list-style-type: none"> carries through short responses 	4a, 14d	<ul style="list-style-type: none"> carries through extended responses 	3, 14e
<ul style="list-style-type: none"> calculates specific cases of generalisations 	13biii	<ul style="list-style-type: none"> generalises mathematical structures 	13cii
<ul style="list-style-type: none"> makes commonsense connections 	13a	<ul style="list-style-type: none"> makes counterintuitive connections 	4b, 4c
<ul style="list-style-type: none"> calculates for a given set of conditions 	14b	<ul style="list-style-type: none"> determines the effects of changed conditions 	4c
<ul style="list-style-type: none"> applies mathematical methods in practised ways 	4b, 11b, 14b	<ul style="list-style-type: none"> applies mathematical methods in unpractised ways 	14a
<ul style="list-style-type: none"> translates between representations in practised ways 	14c, 14d	<ul style="list-style-type: none"> translates between representations in unpractised ways 	14e
<ul style="list-style-type: none"> interprets in practised ways 	4a, 13a	<ul style="list-style-type: none"> interprets in unpractised ways 	
		<ul style="list-style-type: none"> combines interpretation of multiple representations 	4c, 13bii, 13cii, 14e
<ul style="list-style-type: none"> uses a calculator to produce statistics 	11a, 13bii, 14b		
<ul style="list-style-type: none"> produces and copies calculator graphs to show basic features 	3	<ul style="list-style-type: none"> produces and copies calculator graphs to show obscure or unusual features 	
		<ul style="list-style-type: none"> determines boundary values 	3
obeys conventions and attends to accuracy		obeys conventions and attends to accuracy	
<ul style="list-style-type: none"> defines introduced variables 	13bii		
<ul style="list-style-type: none"> applies conventions for diagrams and graphs 	3		
<ul style="list-style-type: none"> uses the equals sign correctly when introducing new operations 		<ul style="list-style-type: none"> uses inequality signs correctly 	3
<ul style="list-style-type: none"> correctly enters data into a calculator, bracketing terms appropriately 	11a, 14b		
<ul style="list-style-type: none"> rounds to suit contexts and specified accuracies in short responses 	4b	<ul style="list-style-type: none"> rounds to suit contexts and specified accuracies in extended responses, and boundary values 	3, 14e
links mathematical results to data and contexts to reach reasonable conclusions		links mathematical results to data and contexts to reach reasonable conclusions	
<ul style="list-style-type: none"> attends to units in short responses 	11a, 11b	<ul style="list-style-type: none"> attends to units in extended responses 	
<ul style="list-style-type: none"> recognises specified conditions in short responses 		<ul style="list-style-type: none"> recognises specified conditions in extended responses 	3
		<ul style="list-style-type: none"> determines limitations of models 	13biii, 13ci
<ul style="list-style-type: none"> interprets single-dimension quantities in terms of contexts 		<ul style="list-style-type: none"> interprets multiple-dimension quantities such as gradient in terms of contexts 	13bii
communicates mathematical reasoning, results and conclusions		communicates mathematical reasoning, results and conclusions	
<ul style="list-style-type: none"> shows working, including expressions entered into a calculator 	4b, 14b	<ul style="list-style-type: none"> shows main steps in reasoning 	3
<ul style="list-style-type: none"> justifies or explains with a simple or routine statement 	4a, 14c	<ul style="list-style-type: none"> justifies or explains by stating methods or properties that have been applied 	14a
		<ul style="list-style-type: none"> meets algebraic and geometric proof requirements 	14a

* ambiguity that might arise because of incomplete or naive interpretation

Note: instances of the task characteristics, in addition to those listed above are identified in the sample tasks.

Sample tasks: source and description

The sample tasks are from the 2005 Applicable Mathematics Tertiary Entrance Examination. The tasks chosen effectively discriminate between students and match the syllabuses for 3AMAT and 3BMAT.

The marking schemes are reported with the tasks. The marking schemes can be seen to reward the *mathematical behaviours* that are embedded in the grade descriptors. The behaviours are that the student:

- identifies and organises relevant information
- chooses effective models and methods and carries through methods correctly
- obeys conventions and attends to accuracy
- links mathematical results to data and contexts to reach reasonable conclusions
- communicates mathematical reasoning, results and conclusions.

The *mathematical behaviours* demanded by each sample task are summarised after the marking scheme. A general description of each behaviour is provided, for example, *identifies and organises relevant information*, then, the specific form of the behaviour is stated, for example, *identifies relevant information from dense information*. These behaviours are characterised as 'simple' or 'deals with complexity'.

In addition, students' answers that do not demonstrate the necessary behaviours are listed, and the behaviours in deficit are identified, for example, *the student did not carry through deductive reasoning*. This deficit approach highlights how marking schemes before the implementation of the new courses have recognised all the behaviours that underpin the new grade descriptors.

Note:

- the analysis of the tasks was informed by a statistical analysis of students' scores and by inspection of students' scripts.
- teachers are not expected to do a detailed analysis of tasks in the manner shown here. The analysis is included to illustrate the behaviours that are specified in descriptors.
- all tasks assume the use of graphics calculators. The availability of CAS calculators would not impact the sample tasks here.
- the judgements of 'simple' and 'deals with complexity' were based on students' responses in the TEE examination. With school-based assessments these judgements could vary between schools, according to the extent to which methods are practised in class prior to an assessment.

3. (5 marks)

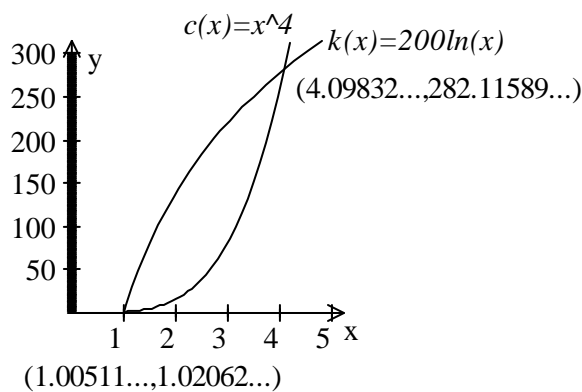
Chris and Kim each have flour-manufacturing companies. The company owned by Chris returns an hourly profit that can be modelled as $c(x) = x^4$, while the company owned by Kim returns an hourly profit that can be modelled as $k(x) = 200 \ln(x)$, where, in each case, x is the amount (in hundreds of kilograms) of flour produced per hour.

In addition, each company must produce at least one hundred kilograms of flour per hour and they can produce no more than four hundred kilograms of flour per hour. If the two companies manufacture the same amount of flour per hour, give the range of production (in terms of x) where the company owned by Kim makes a larger profit than the company owned by Chris.

Indicate clearly the method you used to answer this question and give your answer correct to four decimal places.

Solution

Require $x^4 < 200 \ln(x)$ on $1 \leq x \leq 4$



Graph and find points of intersection

$(1.005116154, 1.0206222026)$

$(4.0983296724, 282.11589874)$,

then look and see where inequality is satisfied.

✓✓(working well explained)

$$1.0051 < x \leq 4$$

$$\text{or } x \in (1.0051, 4]$$

✓✓(one mark each number)

✓ (correct interval)

Also possible to solve equations and use test values to find correct region.

continued next page

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
graph $c(x) = x^4$ $k(x) = 200 \ln(x)$ locate points of intersection (1.00511, 1.02062) (4.09832, 282.11589) determine range within the boundary values 1 and 4 and round to four decimal places $1.0051 < x \leq 4.0000$	identifies and organises relevant information	identifies information from dense information (deals with complexity).
	chooses an effective method and correctly carries through the method	solves an unstructured problem using sub-problems (solves the equations simultaneously, applies the boundary conditions) (deals with complexity). produces and copies calculator graphs to show basic features (intersection points) (simple). carries through deductive reasoning (deals with complexity). carries through an extended response (deals with complexity). determines boundary values (deals with complexity).
	obeys conventions and attends to accuracy	applies conventions for graphs (labels the axes and the curves, not rewarded in the above marking scheme) (simple). uses inequality signs correctly (deals with complexity). rounds to suit a specified accuracy in an extended response (4 decimal places) (deals with complexity). rounds to suit boundary values (deals with complexity).
	links mathematical results to data	recognises specified conditions in extended responses ($1 \leq x \leq 4$) (deals with complexity).
	communicates mathematical reasoning	shows main steps in reasoning (sketches the graph, records the intersection points, records the domain $1 \leq x \leq 4$) (deals with complexity).

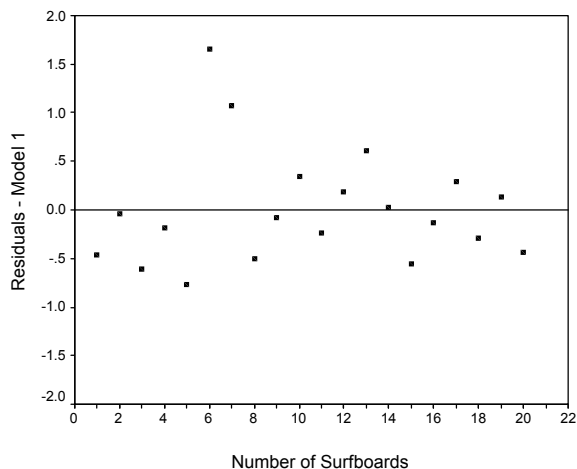
Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

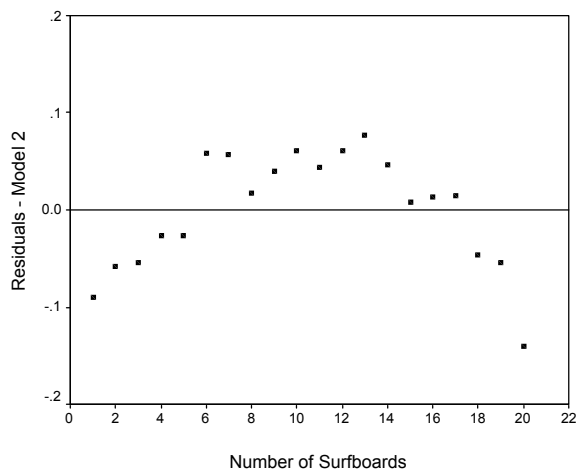
	Behaviour in deficit	Specific omission
$100 \leq x \leq 400$	identifies relevant information	does not correctly identify units for x (x is the number of hundreds of kilograms)
$1.005 \leq x \leq 4.0983$	identifies relevant information/attends to accuracy	does not identify the upper bound of x is 4
$1 > x > 4$	obeys conventions	does not use inequality signs correctly
	attends to accuracy/links mathematical results to data	does not answer to four decimal places as specified does not round to take into account boundary values
graphs sketched without explanation	communicates mathematical reasoning	does not show the method as required

4. (6 marks)

The two residual plots given below were obtained from fitting linear models to the original data and to the data after it had been transformed. The data represents the set-up cost, c (in dollars per item), for a small manufacturing firm making x surfboards.



Model 1: $c(x) = -1.422x + 41.879$



Model 2: $c(x) = -0.056 \ln x + 3.835$

(a) Briefly justifying your answer, indicate which model should be used for predicting the set-up cost per surfboard. [2 marks]

Solution

Model 1 ✓

The residuals are more random ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (the graphs and models) (simple).
	chooses an effective method and correctly carries through the method	interprets in practised ways (simple). carries through a single thread of reasoning (simple). carries through a short response (simple).
	communicates mathematical reasoning, results and conclusions	justifies with a simple or routine statement (the residuals are more random) (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
Model 2 because there is a pattern in Model 2 but not in Model 1	chooses an effective method	does not refer to the residuals
Model 1 – it would have a greater coefficient	chooses an effective method	does not refer to randomness or distribution of the residuals

(b) Using the model you chose in (a), predict the total cost of setting up for manufacturing 9 surfboards. [2 marks]

Solution

$$\checkmark \quad \$ 9 \times (-1.422 \times 9 + 41.879) \quad \checkmark \quad \{ = \$9 \times 29.081 = \$261.73 \}$$

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
$9 \times (-1.422 \times 9 + 1.879)$ = \$261.729 Total set-up cost for 9 boards = \$261.73	identifies and organises relevant information	identifies information in dense information (at the start of the question—identifies that the set-up cost \$c is for <i>each</i> board) (deals with complexity).
	chooses an effective method and correctly carries through the method	applies mathematical methods in practised ways (substitutes into a provided model) (simple). makes a counterintuitive connection (that set-up cost \$c is for each board and not for all boards being produced) (deals with complexity).
	attends to accuracy	rounds to suit contexts in short responses (\$ answers to 2 decimal places) (simple).
	communicates mathematical reasoning	shows working, including expressions entered into a calculator (simple).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
set-up cost = $-1.422 \times 9 + 41.879$ = \$29.08	identifies relevant information	does not identify that Model 1 gives the set-up cost for one surfboard
	links mathematical results to data to reach a reasonable conclusion	does not consider the reasonableness of the response as a set-up cost

- (c) Unfortunately, when the data was recorded an error was made in writing down one of the set-up costs. The actual cost was lower than the recorded value. If the first model is appropriate, circle the residual belonging to the most likely erroneous value and indicate what correcting this value would do to the correlation coefficient. [2 marks]

Solution

See graph for point ✓

Correlation coefficient decreases i.e. gets closer to -1 ✓

{increase in absolute value}

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies the meaning of a single relative term (lower than) (simple).
	chooses an effective method and correctly carries through the method / links mathematical results to data to reach a reasonable conclusion	<p>determines the effects of changed conditions (deals with complexity).</p> <p>carries through deductive reasoning (reasons which residual point to choose, visualises the actual point in the context of Model 1, reasons about the effects on the correlation coefficient while recognising Model 1 has a negative slope) (deals with complexity).</p> <p>combines interpretation of multiple representations (the residual plot and equation for Model 1, in particular the negative gradient) (deals with complexity).</p> <p>makes a counterintuitive connection (links <i>stronger</i> correlation with a <i>decrease</i> in the correlation coefficient) (deals with complexity).</p>

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
correctly circles the point furthest above the line	links mathematical results to data to reach a reasonable conclusion	does not link correlation with the negative slope of the linear model
correlation coefficient increases		

11. (9 marks)

The failure times, to the nearest minute, for a sample of twenty overloaded computer components are given below.

32 25 47 26 33 35 29 45 27 16
 26 30 18 34 71 32 55 51 80 67

(a) Calculate the mean and standard deviation for these failure times. [2 marks]

Solution

mean = 38.95 minutes ✓

standard deviation = 17.31 minutes ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple).
	chooses an effective method and correctly carries through the method	uses a calculator to produce statistics (simple). carries through a short response (simple).
	attends to accuracy	correctly enters data into a calculator (simple). rounds to suit contexts in short responses (mean to one decimal place, not rewarded in the above marking scheme) (simple).
	links mathematical results to data and contexts	attends to units in short responses (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
$\bar{x} = 37.45$ $S_x = 18.98$	attends to accuracy	does not correctly enter data into a calculator
	links mathematical results to data	does not include units (minutes)

(b) Suppose that failure times (to the nearest minute) are available, from an extended study, for 300 overloaded computer components. These failure times range from 16 minutes to 84 minutes and are summarised in the table below.

Failure times (minutes)	Cumulative Frequency
15 – 24	47
25 – 34	175
35 – 44	226
45 – 54	256
55 – 64	266
65 – 74	286
75 – 84	300

For these grouped data find:

(i) the mean and standard deviation of the failure times.

[3 marks]

Solution

mean = 37.63 minutes ✓✓

standard deviation = 16.12 minutes ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (in particular identifies that the table shows cumulative frequency) (simple).
	chooses an effective method and correctly carries through the method	applies mathematical methods in practised ways (calculates the midpoints of the intervals and the frequencies) (simple). uses a calculator to produce statistics (simple).
	attends to accuracy	correctly enters data into a calculator (simple).
	links mathematical results to data and contexts	attends to units in short responses (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
$\bar{x} = 58.50$ $S_x = 17.51$	identifies and organises relevant information	does not identify that provided frequencies are cumulative (the answers have been calculated with cumulative frequencies)
	links mathematical results to data and contexts	does not include units in the answers
$\bar{x} = 38.13$ minutes $S_x = 16.62$ minutes	carries through the method correctly	does not correctly calculate midpoints for the intervals (used 20, 30, 40 ... as midpoints)

(ii) the proportional median failure time.

[3 marks]

Proportional median is not included in 3AMAT or 3BMAT

(iii) the largest possible number of components with a failure time less than 27 minutes, given that for the 300 individual data items the lower quartile was 26.5 minutes.

[1 mark]

Solution

75 ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	interprets multiple relative terms (largest, less than, lower (deals with complexity).
	chooses an effective method and correctly carries through the method	carries through deductive reasoning (reasons that 75 data items lie below the lower quartile, data is to the nearest minute, 75 th item must be 26 minutes or less, 76 th must be 27 minutes or more, concludes the largest possible number of components is 75) (deals with complexity).

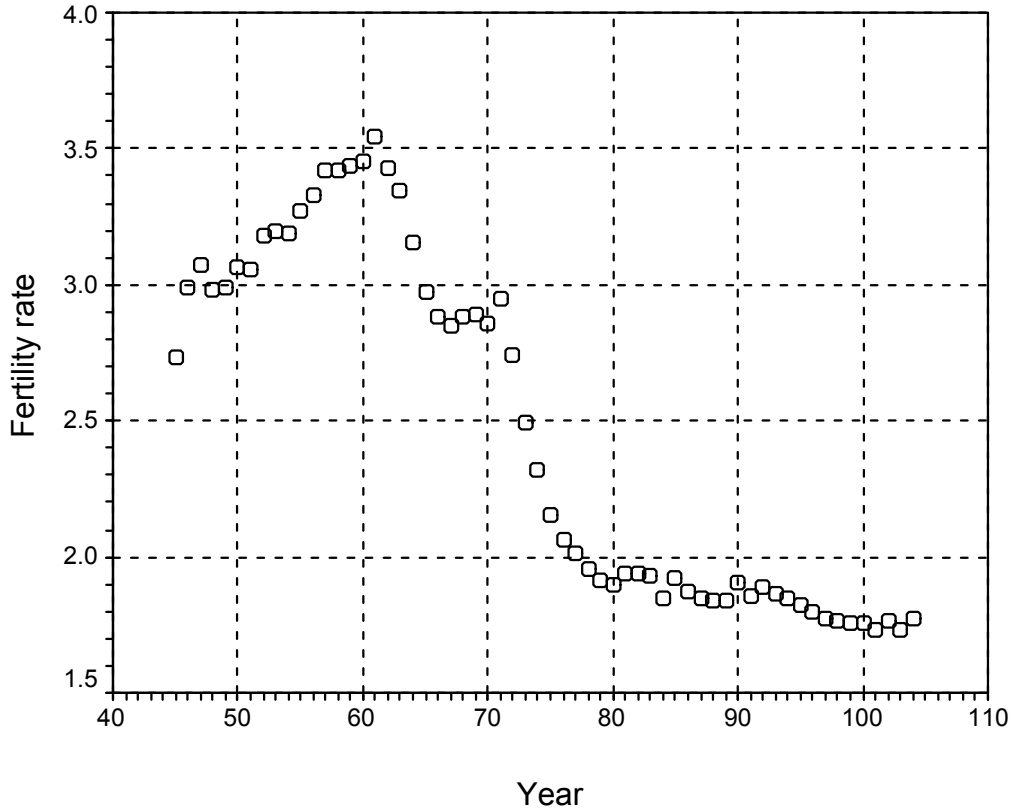
Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
26.5	identifies and organises relevant information	does not identify that a <i>number</i> of components is required, not a time (26.5 minutes)
74	carries through the method correctly	errors in reasoning

13. (14 marks)

The fertility rate of a country is the mean number of children that a woman in that country is likely to bear during her lifetime. The fertility rate, $F(t)$, for Australia (as published in *The Weekend Australian*, April 9-10 2005) between the years 1945 and 2004 is shown in the graph below, with 45 representing the year 1945, 46 representing 1946 and so on to 104 representing 2004. Some data is also given in the table.



Year	45	46	47	48	49	50	51	52	53	54
Fertility Rate	2.736	2.986	3.076	2.985	2.986	3.067	3.059	3.181	3.193	3.191
Year	55	56	57	58	59	60	61	62	63	64
Fertility Rate	3.275	3.330	3.421	3.421	3.440	3.451	3.548	3.431	3.343	3.154

(a) Briefly describe any apparent trends in the fertility rate for Australia.

[2 marks]

Solution

Increases to about 1961 ✓ then decreases. ✓

continued next page

(Note: if using this question in an assessment, fertility rates are published by the Australian Bureau of Statistics and are for the mean number of babies born per woman, not the number of children a woman is likely to bear. For example see

<http://www.abs.gov.au/AUSSTATS/abs@.nsf/7d12b0f6763c78caca257061001cc588/D780E4455F70CFE5CA2573D200110123?opendocument>)

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple). recognises a question has more than one answer from the plural noun ('trends', plural, is mentioned) (simple).
	chooses an effective method and correctly carries through the method	interprets in potentially practised ways (the two trends on the graph and the vicinity of the change in trend) (simple). makes a commonsense connection (where points on the graph get higher/lower, the fertility rate respectively increases/decreases) (simple) (cf. comparing swimming records over time—the records <i>improve</i> as swimming times <i>decrease</i> which is counterintuitive) carries through a short response (simple)
	links mathematical results to data and contexts	links mathematical results to contexts in short responses (the fertility rate context, not rewarded in the above TEE marking scheme) (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
fertility rate is decreasing as the years increase	identifies and organises relevant information	does not identify that there is more than one trend
throughout approximately 1949 to 1962 there was an exceptionally high fertility rate. The overall trend however has been a downward slope from approximately 3 in 1945 to 1.7 in 2004	identifies and organises relevant information	does not identify that there are two trends
	attends to accuracy	the 'high fertility rate' claimed for 1949-1962 is also evident in the 'high' figures during 1963-1971 but these years are not mentioned

(b) It is possible to represent the trend in the early years using a line of best fit.

(i) Give an appropriate year to use as the final year for such calculations.

[1 mark]

Solution

1961 ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	chooses an effective method and correctly carries through the method	interprets in potentially unpractised ways (identifies precisely the year when one trend finishes and the other starts) (deals with complexity). carries through deductive reasoning (a trend describes change, the changes in fertility rate are positive up to 1961) (deals with complexity).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
1964	identifies relevant information	does not identify that the last year in the table, 1964, is not part of the early trend
1960	chooses an effective method	does not deduce that the increasing trend stops in the same year (1961) as the decreasing trend starts

(ii) Calculate an appropriate least squares regression line for these early years and indicate what the slope of the regression line represents. [3 marks]

Solution

$$\text{fertility rate} = 0.9773 + 0.0419 \times \text{year}$$

increase in fertility rate {by 0.042} per year

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	chooses an effective method and correctly carries through the method	uses a calculator to produce statistics (the regression line) (simple). combines interpretation of multiple representations (the regression equation and the graph or table, in order to decide what the slope of the regression line represents) (deals with complexity).
	obeys conventions and attends to accuracy	enters data correctly into a calculator (simple). defines introduced variables (if the line is written as $y = 0.0419x + 0.9773$) (simple).
	links mathematical results to data and contexts	interprets multiple-dimension quantities in terms of contexts (the slope or gradient) (deals with complexity).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
$y = 1.602 + .02961x$ (the equation for the whole table of data)	identifies relevant information	does not identify to use some data only from the table
an increasing slope is positive	links mathematical results to data and contexts to reach reasonable conclusions	does not link the slope of the regression line to the context

(iii) Use your equation to predict the fertility rate for 1970 and plot this value clearly on the graph on the previous page. Comment upon your value. [4 marks]

Solution

fertility rate = 3.91 ✓ plot on graph ✓

Predicted value much higher than the observed value ✓

The linear model is not appropriate in 1970 {extrapolation} ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple).
	chooses an effective method and correctly carries through the method	calculates specific cases of a generalisation (simple).
	links mathematical results to data and contexts	determines limitations of models (the regression model summarises the trend until 1961, extrapolation to 1970 is not appropriate) (deals with complexity).

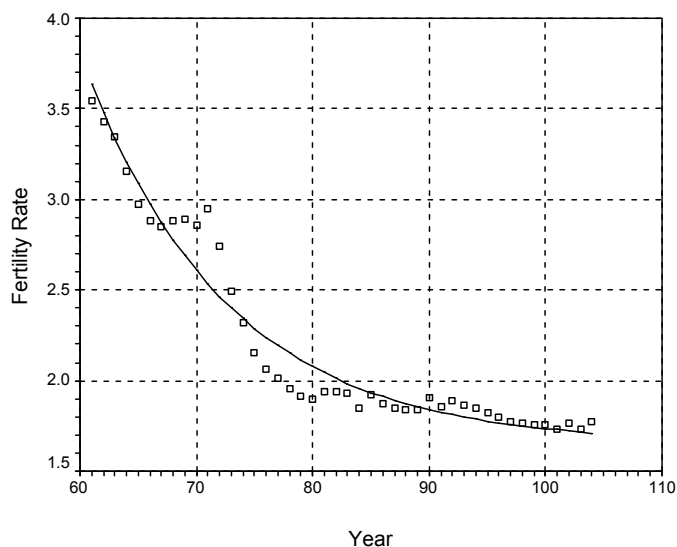
Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
(70, 3.91) plotted Comment: It is not as close to the given number in the graph as it should be. The actual value is lower so the prediction is extrapolating.	links mathematical results to data and contexts	does not recognise the model summarises the trend until 1961 only

It is possible to represent the fertility rate in the later years using an exponential model. The graph below indicates the fit of the data to the model

$$F(t) = 277.651e^{-0.081t} + 1.65 \quad 61 \leq t \leq 104.$$



(c) (i) Comment upon the fit of the data to the model.

[2 marks]

Solution

On the whole not bad ✓

but could be better between approx 1967 and 1981 ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	links mathematical results to data and contexts	determines limitations of models (deals with complexity).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
The data fits the model very well so there will be a high correlation coefficient	links mathematical results to data	erroneously mentions the correlation coefficient which measures fit for linear models.
	communicates mathematical reasoning	does not explain why the data 'fits the model very well'

(ii) Give a reason for the " $+ 1.65$ " term in the model.

[2 marks]

Solution

The data is asymptotic ✓ towards 1.65 ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	chooses an effective method and correctly carries through the method	combines interpretation of multiple representations (the equation and the graph) (deals with complexity). generalises a mathematical structure (the asymptotic behaviour) (deals with complexity).

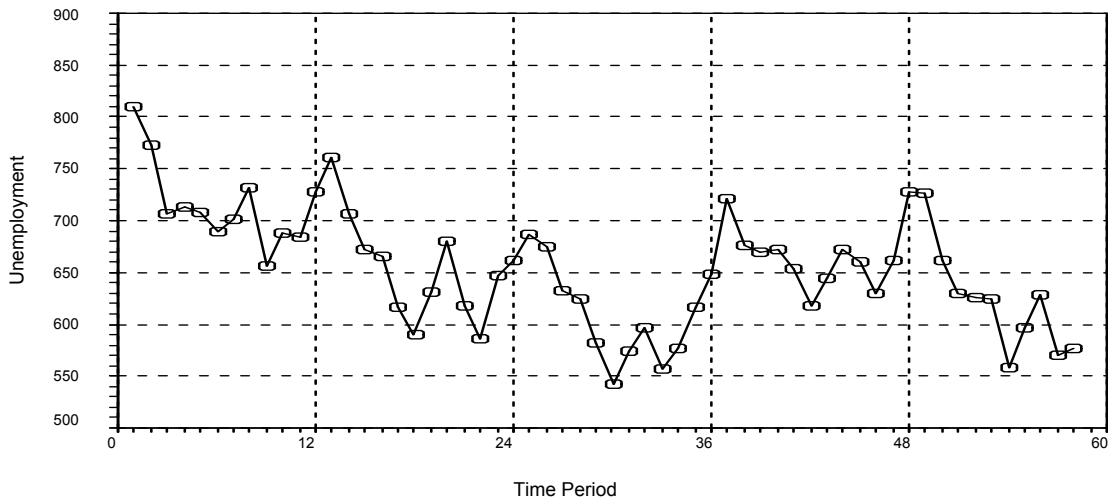
Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
1.65 is the initial fertility rate	chooses an effective method and correctly carries through the method	does not interpret the equation or the graph correctly

14. (9 marks)

Monthly data relating to the number of unemployed people (in thousands) in Australia was collected for the period February 1998 to November 2002. (Data courtesy of the Australian Bureau of Statistics) A graph of this data is given below with February 1998 represented by time period $t = 1$, March 1998 represented by $t = 2$, ..., November 2002 represented by $t = 58$.



(a) For this data, explain why a 12-point centred moving average is more appropriate than a 6-point centred moving average. [1 mark]

Solution

From graph, pattern repeats every 12 months ✓

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
From graph, peaks occur after 7 then 5 months. A 12 month cycles takes both these in whereas a 6 month cycle does not. or from the graph, there is a peak every 12 months and there is a trough 6 months after each peak. A cycle goes from peak to peak, so a 12 month cycle is more appropriate than a 6 month one.	identifies and organises relevant information	identifies key elements in ambiguous data (identifies a cycle in data that does not follow the usual peak, trough, peak pattern) (deals with complexity)
	chooses an effective method and correctly carries through the method	applies a mathematical method in unpractised ways (counts points in a cycle when the cycle is not distinct) (deals with complexity).
	communicates mathematical reasoning, results and conclusions	explains by stating methods or properties that have been applied (deals with complexity). meets algebraic and geometric proof requirements (explains why the 12-point average is appropriate and why the 6-point average is not appropriate—not required in the above TEE marking scheme) (deals with complexity).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
There are 12 months in a year	identifies and organises relevant information	does not identify that moving averages are to be considered
A 12-point moving average smooths out a greater range and takes into account more fluctuations	chooses an effective method	does not consider how many points (6 or 12) form a cycle

Nov-1998	10	688.7
Dec-1998	11	684.6
Jan-1999	12	727.9
Feb-1999	13	760.4
Mar-1999	14	706.7
Apr-1999	15	672.1
May-1999	16	665.5
Jun-1999	17	616.2
Jul-1999	18	590.4
Aug-1999	19	630.8
Sep-1999	20	679.8
Oct-1999	21	618.4
Nov-1999	22	586.3
Dec-1999	23	646.9
Jan-2000	24	661.0
Feb-2000	25	686.8
Mar-2000	26	674.7
Apr-2000	27	632.0
May-2000	28	624.0
Jun-2000	29	582.4

(b) Some of the original unemployment data is given in the table on the left. Calculate the 12-point centred moving average for July 1999 ($t = 18$).

[2 marks]

Solution

$$\frac{7867.95}{12} = \{655.66\} \checkmark\checkmark$$

$$\left\{ \begin{array}{l} (363.95 + 760.4 + 706.7 + \\ 672.1 + 665.5 + 616.2 + \\ 590.4 + 630.8 + 679.8 + \\ 618.4 + 586.3 + 646.9 + \\ 330.5) / 12 \end{array} \right\}$$

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
$\frac{363.95 + 760.4 + \dots + 330.5}{12}$ $= \frac{7867.95}{12}$ ≈ 655.66 <p>(moving average ≈ 655.7 thousand unemployed people)</p>	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple).
	chooses an effective method and correctly carries through the method	applies mathematical methods in practised ways (calculates the moving average) (simple). calculates for a given set of conditions (simple). carries through a single thread of reasoning (simple). uses a calculator to produce statistics (simple).
	attends to accuracy	correctly enters data into a calculator (simple).
	communicates mathematical reasoning and results	shows working, including expressions entered into a calculator (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
657.3	attends to accuracy	does not correctly enter data into a calculator
658.45	chooses an effective method	does not average the correct 12 points

All possible 12-point centred moving averages were calculated and then used to find the equation of the linear *trend* in the model:

predicted unemployment = trend + seasonal component + random component.

(c) Four possible equations for the *trend* are given below. Which do you consider to be the most appropriate? Justify your answer. (Note: y is in thousands)

$$y = 812 - 0.9t \quad y = 677 - 0.9t \quad y = 677 + 0.3t \quad y = 812 + 0.3t \quad [3 \text{ marks}]$$

Solution

$$y = 677 - 0.9t \checkmark$$

slope must be negative \checkmark

starting at 812 puts the line too high \checkmark

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple).
	chooses an effective method and correctly carries through the method	translates between representations in potentially practised ways (match equations with an imagined trend line on the graph) (simple).
	communicates mathematical reasoning	justifies with routine statements (simple).

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

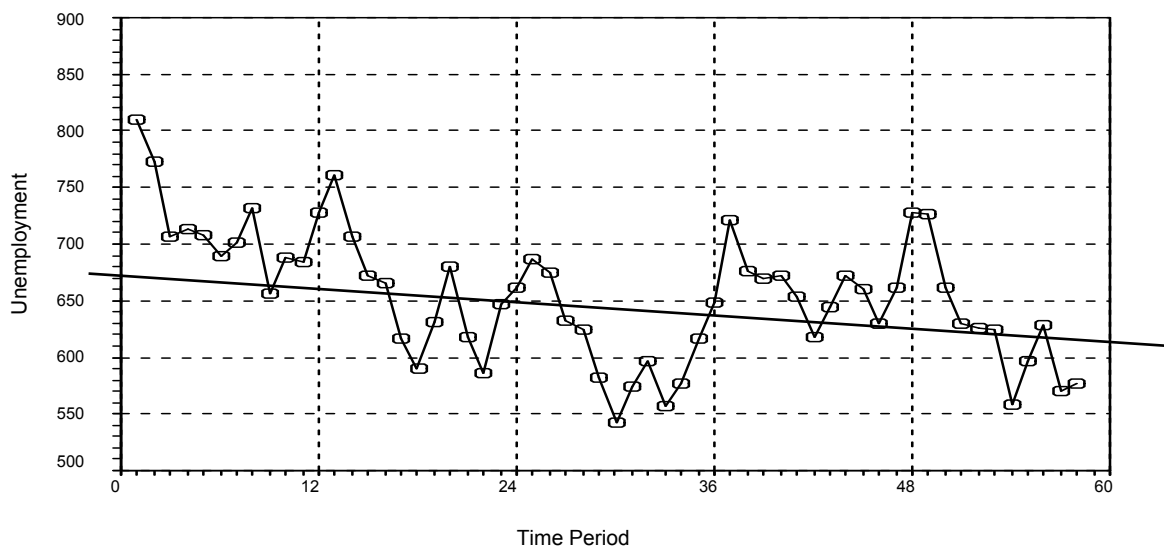
	Behaviour in deficit	Specific omission
$y = 812 - 0.9t$ The trend is negative	carries through methods correctly	does not consider the y intercept

(d) Draw the appropriate trend line on the graph on the previous page.

[1 mark]

Solution

Draw line on graph ✓ {possible points to help plot (0, 677) (60, 623)}



Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
see above	identifies and organises relevant information	identifies relevant information in information that is narrow in scope (simple).
	chooses an effective method and correctly carries through the method	answers a structured question that requires a short response (part (c) lays the ground for part (d)) (simple). translates between representations in practised ways—graphs the linear function) (simple). carries through a short response (simple).
	links mathematical results to data and contexts to reach reasonable conclusions	determines limitations of models (checks the reasonableness of the line after drawing it (deals with complexity, but only necessary if the wrong line is drawn)

Students' TEE performance and the behaviours that are required indicate that responses to this question are evidence for **Grades B-E**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
drew $y = 812 - 0.9t$	links mathematical results to data and contexts to reach a reasonable conclusion	does not consider whether the drawn line was reasonable
$y = 677 - 0.9t$ placed incorrectly on the graph	attends to accuracy	does not accurately plot points

(e) Estimate (to the nearest 50 thousand) the seasonal components for February and July.

[2 marks]

Solution

Estimating from graph, seasonal component for February approximately 100 thousand ✓

{using 1, 13, 25, 37, 49 => $(140+100+40+80+100)/5 = 92$ }

{using 13, 25, 37, 49 => $(100+40+80+100)/4 = 80$ }

Seasonal component for July approximately -50 thousand ✓

{using 6, 18, 30, 42, 54 gives $(20-70-100-20-80)/5 = -50$ }

{using 6, 18, 30, 42 gives $(20-70-100-70)/4 = -42.5$ }

Key behaviours for a correct solution

Solution	General behaviour	Specific form of the behaviour
Seasonal components for February approximately 100 thousand	identifies and organises relevant information	identifies key information from dense information (identifies all relevant data on the graph) (deals with complexity).
using data for time periods 1, 13, 25, 37, 49 $(140+100+40+80+100)/5 = 92$	chooses an effective method and correctly carries through the method	combines interpretation of multiple representations (determines from the table that $t=13$ is February on the graph, $t = 18$ is July on the graph) (deals with complexity). translates between representations in potentially unpractised ways (determines seasonal adjustment values from a graph rather than a table) (deals with complexity). carries through an extended response (deals with complexity).
Seasonal components for July approximately -50 thousand using data for time periods 6, 18, 30, 42, 54 $(20-70-100-20-80)/5 = -50$	obeys conventions and attends to accuracy	correctly enters data into a calculator (simple). rounds to suit specified accuracies in extended responses (deals with complexity).

Analysis of students' TEE performance indicates that success on complex aspects of this question is evidence for **Grades A and B**.

Sample responses—incorrect and partial responses

	Behaviour in deficit	Specific omission
Feb: $\frac{760.4 + 686.8}{2} = 723.6$ People 723.6×5 $= 3618$ $= 3618000$	identifies and organises relevant information	does not use residuals—instead averages the 2 unemployment figures for February from the table

Table 3: Tasks mapped to the syllabus

Question	Concepts	Topic
Number and algebra		
1	3AMAT 1.2.3, 1.3.3	solving simultaneous equations graphically
Chance and data		
4a	3BMAT 3.2.3	residual plot
4b	3BMAT 3.2.8	predicting from regression lines
4c	3BMAT 3.1.2, 3.2.3	outlier, correlation coefficient
11a	3AMAT 3.4.3, 3.4.7	mean and standard deviation
11bi	3AMAT 3.4.4, 3.4.7	mean and standard deviation
11bii	3AMAT 3.4.6	quartiles
13a	prior knowledge	
13bi	3BMAT 3.1.4.	calculating a regression line
13bii	3BMAT 3.1.4	calculating a regression line
13biii	3BMAT 3.2.5	predicting from a regression line
13ci	3BMAT 3.2.4	alternative model (exponential)
13cii	3AMAT 1.2.5	asymptotic behaviour
14a	3BMAT 3.1.6	moving average property
14b	3BMAT 3.1.6	moving average calculation
14c	3BMAT 3.1.6	regression line for cyclical data
14d	3BMAT 3.1.6	graphing a regression line
14e	3BMAT 3.2.8	predicting from a regression line

Note:

- the number of tasks in each category was determined by the correspondence between the mathematics content of 2005 Applicable Mathematics TEE questions and the 3AMAT/3BMAT syllabus and does not reflect emphases in the 3AMAT/3BMAT syllabus
- the questions target particular concepts but, as shown in the analysis of tasks, assessment is based on how students work mathematically in relation to concepts. Hence, it is important to teach 'working mathematically' processes.