

Mathematics HP CAS items

1. Is $(x + 8)^2 - 4$ equivalent to $(x + 7)(x + 9) - 3$ for all values of x ?
You must show mathematical statements to support your answer.

2. A mathematics class is investigating rectangles that each have perimeter 150 cm.

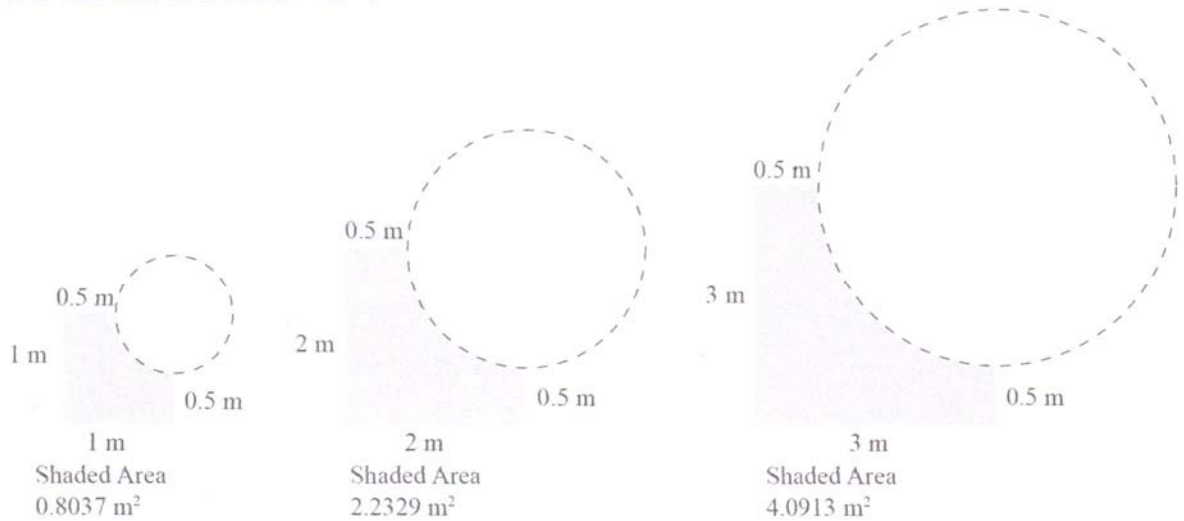
If the perimeter of a rectangle is 150 cm, give the rule for the area, A , in terms of the width, w .

3.

Eli is building a skateboard ramp.

Part of the ramp (called the rib) is shown by the shaded areas in the diagrams below. It is made by cutting a quarter circle out of a square.

Hint: the area of a circle = πr^2 .



Find a rule that describes the relationship between the base length of the square, b , and the area of the rib, A .

Use the rule to find the area of the rib if he uses a 7 m square.

4.

John notices that the solution for both of these pairs of simultaneous equations is the same.

$$\begin{cases} x + 2y = 3 \\ 3x + 2y = 1 \end{cases} \quad \text{and} \quad \begin{cases} 10x + 11y = 12 \\ 4x + 3y = 2 \end{cases}$$

(a) Find the answer to both pairs of equations to show that John is correct.

John also notices that :

- (i) the first equation has consecutive increasing coefficients, and
- (ii) the second equation has consecutive decreasing coefficients.

He wonders if other pairs of equations with coefficients like these also have the same solution.

(b) Form and solve another pair of simultaneous equations that fit this pattern, and see if he is correct. You must give the pair of simultaneous equations that you use.

(c) John wants to know if this is true for all cases.

Write a generalised pair of equations that fit the pattern and use them to solve John's problem. You must show mathematical statements to support your answer.

Further CAS questions

The examples below were taken from tests in Introductory Calculus, Applicable Mathematics and Calculus with the original mark allocation indicated. You may wish to try them using the CAS. As you do, please consider the following points:

- Can you use the CAS to show the process skills involved?
ie. How do you show working?
- How will access to a CAS affect:
 - how you teach this topic?
 - how you assess this topic?

[3,4,3 marks]

Simplify the following giving your answers with positive powers.

(a) $\frac{6a^3b^4}{-4ab^6}$ (b) $\frac{(2x^3y^{-1})^3(3x^2y)^{-2}}{(6x^{-1}y^2)^{-2}}$ (c) $\sqrt{xy} \div \frac{y}{2x}$

[2,2,2,2,3 marks]

Differentiate the following with respect to x giving answers with positive powers.

(a) $y=8x^2-3x+4$ (b) $y=\frac{9}{x}+5\sqrt{x}$ (c) $y=5(2-3x)^2$

(d) $y=(1+x+x^2)(x^2-4)$ [Do not simplify your answer]

(e) $y=\frac{2}{(1-x^2)^2}$

[2,2,3,3,2,2,3 marks]

a) Express as a single logarithm

i) $3\log_a m + 4\log_a n - 5\log_a t$ ii) $(\log x)^3 \div (\log x)^2 + \log x^2$

b) Solve each of the following showing full working:

i) $8x^{\frac{1}{3}} + 12x^{\frac{1}{3}} = 40$ ii) $\log_5 x + \log_2 8 = 0$

c) Evaluate each of the following showing full working:

i) $3\log_2 6 - \log_2 27$ ii) $1.5\log_8 4$

d) Solve $3^{x+1} = 12$ using logarithms and giving your answer to 1 decimal place.

[2 marks]

Find: y if $\frac{dy}{dx} = (3x-2)^5$

[11 marks]

Integrate each of the following with respect to the appropriate variable.

(a) $\int \frac{4x^3 - x + 12}{x^3} dx$ (b) $\int \sin^2 \frac{x}{2} dx$ (c) $\int (2x+5)^4 dx$ (d) $\int \sin x \cos^5 x dx$

[15 marks]

Evaluate exactly where possible

- (a) $\int_2^3 \frac{x}{\sqrt{x+1}} dx$ Let $u = x + 1$ (b) $\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}} dx$ Let $3x = 2\sin\theta$
(c) $\int_0^2 (\sin x + \cos x)^2 dx$

[2 marks]

Given $(x-2)$ is a factor of $x^4 - ax^3 + ax^2 + ax - 6$, find a .

[2, 2, 2,2 marks]

Evaluate the following limits (if they exist).

- a) $\lim_{x \rightarrow 3} \frac{x^3 - x^2 - x - 2}{x - 2}$ b) $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x - 5}{4x^2 + 4x}$
c) $\lim_{x \rightarrow t} \frac{x^2 - t^2}{x - t}$ d) $\lim_{x \rightarrow 2^+} \frac{\text{int}(x)}{x}$

[5 marks]

Find the values of a and b in the following function if it is continuous and differentiable at $x = 2$.

$$f(x) = \begin{cases} ax^2 + b & x \leq 2 \\ x^3 - 6x + 14 & x > 2 \end{cases}$$

[6,2 marks]

For the system of equations

$$\left. \begin{aligned} 2x - 3y + z &= 4 \\ x - y + kz &= 3 \\ x - 2y + 2z &= k^2 \end{aligned} \right\}$$

calculate the value(s) of k for which the system has:

- (a) an infinite number of solutions. (b) a unique solution