



EXAMINERS' REPORT ON 2001 TERTIARY ENTRANCE EXAMINATION

SUBJECT: DISCRETE MATHEMATICS

STATISTICS

Year	Number Who Sat	Non-Examination Candidates	Did Not Sit
2001	7215	519	723
2000	6695	543	872
1999	6240	440	660

The Examiners' Report is written by the Chief Examiner (or another examiner on their behalf) to comment on matters relating to the Tertiary Entrance Examination in their subject. The opinions and recommendations expressed in this report are those of the Chief Examiner and not necessarily representative of or endorsed by the Curriculum Council.

The Marking Guide provided at the end of this report was prepared for markers and substantially amplified by discussions held in the pre-marking meeting. It is not intended as a set of model answers, and is not exhaustive as regards alternative answers. Some of the answers are less than perfect, but represent a standard of response that the examiners deemed sufficient to earn full marks. Teachers who use this guide should do so with its original purpose in mind.

SUMMARY/ABSTRACT

The structure of the exam was similar to 2000 so should have held no surprises for the candidates. The length of the paper meant that most candidates had time to complete it. An overall mean of 52% shows that the paper was more difficult than the 2000 paper.

The two-way table with probabilities was very similar to 2000, but the idea of comparing proportions instead of actual numbers was still not done well. Rounding in general is not done well especially in the question with mm. Questions that require explanations were generally not done well. Candidates may have known what they meant, but had trouble expressing it in words. Candidates often gave wrong answers with no working so they forfeited any possible follow through marks.

The paper attempted to test the investigation component of the course. This inevitably meant non-routine questions that the less able candidates found difficult.

GENERAL COMMENTS

There were some comments from markers that on the first reading of the paper that it seemed too difficult, but on closer inspection, there were sufficient routine questions for candidates to obtain marks. These included (3) maximum flow, (5) tree diagram, (6) arithmetic series, (8) time series and (12) critical path; all of which had a mean of over 70%.

Questions that candidates found particularly difficult (average under 40%) were (1) sum to infinity, (4) & (11) recursive definitions, (17) Venn diagram and probability with prime, Fibonacci, cubic and square numbers, (20) investigating patterns and finding a formula and (21) finding factors.

The balance of the exam followed the syllabus assessment structures.

The strengths and weaknesses of candidates are similar to last year with prime numbers, factors, probability, rounding and explanations still needing to be improved. It was pleasing that most candidates did show working in (3a), where it was specifically requested. Where working is not specified many candidates tend to give an answer only even when a question is worth say 4 marks, thus risking obtaining zero for the question.

COMMENTS ON SPECIFIC QUESTIONS

1 worth 3 marks attempted by 6762 candidates mean mark = 32.51%

Part (i) was done well, but very few candidates managed part (ii), the most common response being 19/100. This was despite the word *hence* and part (i) clearly mentioning a geometric series and an infinite series.

2 worth 6 marks attempted by 7071 candidates mean mark = 64.56%

- (a) OK
- (b) Main errors were using 184.5 instead of the new mean (minor error) or $(176.5+200)/2$ (major error).
- (c) OK

3 worth 6 marks attempted by 7133 candidates mean mark = 70.31%

- (a) Working shown by most candidates so they gained 2/3 marks even if they did not get the correct total. The most efficient way was not followed by a large number of candidates as 44 was a common total.
- (b) & (c) OK

4 worth 7 marks attempted by 6944 candidates mean mark = 39.75%

- (a) Many who had (i) correct still had difficulty writing $T(4)$ etc in terms of $T(1)$ only. To gain a mark, we wanted $T(4) = -T(1)$ NOT something like $T(4) = T(1) - 10$.
- (b) Even those who had (a)i correct did not necessarily get this right.
- (c) Poorly done.

5 worth 9 marks attempted by 7186 candidates mean mark = 79.38%

- (a) Most common error was not reading the question and allowing repetition.
- (b) Conditional probability caused the most problem. Some candidates wasted time in simplifying. This is counterproductive especially for conditional probability as generally there is a mark for the numerator and a mark for the denominator. If simplification has occurred with an incorrect answer, then a candidate cannot gain part marks for a correct numerator or denominator. Follow through from (a) was allowed and this question had the highest percentage mean and was one of the questions attempted by practically all candidates.

6 worth 7 marks attempted by 7180 candidates mean mark = 74.52%

Generally well done although written explanations for (c) caused some difficulty.

7 worth 10 marks attempted by 7197 candidates mean mark = 59.43%

- (a) OK
- (b) Not done well even though the question was very similar to the previous two years.
- (c) Could not be answered unless (b) was answered correctly. Most candidates spoke about the samples being a different size instead of looking for the proportions.

8 worth 14 marks attempted by 7188 candidates mean mark = 70.78%

- (a) OK
- (b) Generally OK.
- (c) Problems with finding a CMA plus rounding of the result.
- (d) Most candidates did not add the average residual. Many did not show any working even though it was worth 4 marks. Some gave an answer of \$4 with no working. *Maybe* they had done the question correctly but rounded incorrectly at the end in which case they would get 3 out of 4 with working, but no working – no marks.

9 worth 9 marks attempted by 6858 candidates mean mark = 51.14%

(a) Generally OK although some had problems reading the scale and some did not show how they obtained the answer from the graph.

(b) The time interval of 10 caused problems so a common answer was $200(0.75)^t$.

(c) Answer correct to nearest minute caused difficulties for those who answered this question. Either they had rounded values earlier in the question so final answer not accurate or the conversion to minutes was done incorrectly.

10 worth 8 marks attempted by 6564 candidates mean mark = 67.53%

The strange rounding in the last column in the table did not seem to bother the candidates.

(a),(b),(c) done well although some used compound interest formula in (b). In (d), candidates had great difficulty with the explanation. Many spoke about compound interest.

11 worth 8 marks attempted by 6569 candidates mean mark = 21.58%

(a) OK

(b) Very few candidates came up with a formula.

(c) AP was a very common response.

(d) Stating that one of the first two terms must be negative was a common response.

12 worth 8 marks attempted by 7139 candidates mean mark = 78.44%

Even though it was a complicated diagram, the majority of candidates handled it well. Whole question generally well answered. Main problem was with realizing there was a new critical path for (c).

13 worth 4 marks attempted by 7078 candidates mean mark = 63.32%

(a) Many candidates referred to the correlation coefficient.

(b) OK

(c) Some candidates substituted in $x = 30$. A few tried reading from the graph, others rounded incorrectly.

14 worth 13 marks attempted by 7049 candidates mean mark = 44.81%

(a),(b),(c) done reasonably well. The main problem was either that candidates do not know how to round correctly OR they do not know how many mm in a m.

(d) Very few could work out the area of the circle. The first two lines of the table were often attempted but very few even attempted the last line in the table.

15 worth 10 marks attempted by 6905 candidates mean mark = 65.82%

(a) Generally OK.

(b) Common mistakes were $y \geq x$ or $y = x$ instead of $x \geq y$, and $x \geq 0$ instead of $y \geq 0$.

(c) Some candidates thought it was the product of the coordinates though others thought (8,8) was automatically the maximum. Most appeared to know what to do even though some of their explanations were not very clear.

16 worth 7 marks attempted by 7033 candidates mean mark = 59.80%

(a) Most candidates made an attempt, but not many gained full marks. Some ignored the direction and had routes starting ABC. Others did not get any of the longer routes.

(b) Candidates confused between "length" and "demand". The longest route actually did have the highest demand for (i) but in (ii), the majority of candidates gave an arbitrary short route or the short route with the lowest demand!

17 worth 11 marks attempted by 6933 candidates mean mark = 37.41%

(a) Many errors. Did candidates not know the notation or were they unable to recognize prime, Fibonacci numbers etc?

(b) Very poorly done. Some of the good responses left out zero. Some of the candidates are still listing the elements even though the **number** of elements was written in bold.

18 worth 10 marks attempted by 7091 candidates mean mark = 51.94%

- (a) Too many candidates thought it was a linear relationship.
- (b),(c) Follow through allowed candidates to gain marks.
- (d) Cost per wash concept for comparison was not grasped by a large number of candidates.

19 worth 13 marks attempted by 7003 candidates mean mark = 55.14%

- (a) OK
- (b) Many candidates failed to round up.
- (c) Many failed to adjust the number of sales (could have come back and done this after seeing the formula for (d)).
- (d) Even though the formula was given and therefore only calculator skills were needed, there were still some weird answers.
- (e) 4 marks and again some candidates giving an answer with no working. Question often not completed with candidates stopping at $x = 27$.

20 worth 7 marks attempted by 6852 candidates mean mark = 30.64%

- (a) OK
- (b) A common response was 10. (You can make the 10th term if you dismantle all the others which is not what the question was asking).
- (c) Very few correct.

21 worth 10 marks attempted by 5756 candidates mean mark = 11.88%

- (a) Often not systematic in listing the factors.
- (b) (c) Done very poorly. The factors of 500 in particular should have been quite easy to list and count without having to write down the rule.

POINTS FOR CONSIDERATION BY THE SYLLABUS COMMITTEE

Nil

Pamela Hollis
December 2001

2001 Examining Panel

Chief Examiner: Dr David McDougall
Deputy: Mrs Pamela Hollis
Third Member: Mr Christopher Fraser

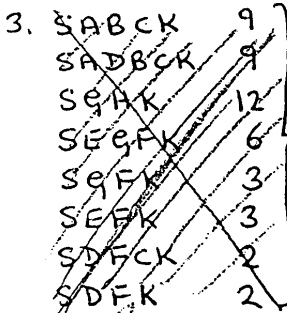
Chief Marker: Mrs Pamela Hollis

1(a) $0.01 \checkmark$
 (b) $\frac{0.19}{0.99} \checkmark = \frac{19}{99} \checkmark$

2(a) $176.5 \text{ cm} \checkmark$
 $6 \text{ cm} \checkmark$

(b) $\frac{(176.5 \times 20 + 200)}{21} \checkmark$
 $= 177.62 \text{ cm} \checkmark$
 (c) Increase \checkmark

$(184.5 \times 20 + 200)$
 21
 $= 185.24$
 1 mark with working
 142.88



SABCK	9	SABCK	9
SADBCK	9	SADBCK	9
SDBCK	2	SDBCK	2
SDFK	5	SDFK	5
SEFK	6	SEFK	6
SEFK	3	SEFK	3
SEFK	3	SEFK	3
SDFCK	2	SDFCK	2
SDFK	2	SDFK	2
<hr/>		<hr/>	
FC	✓	FC or EQ	no change
no change ✓		no change	

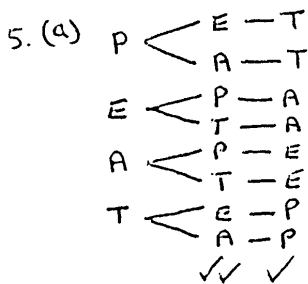
OR on diagram
 Total 46 ✓

(b) EF ✓
 (c) $(46 - 3 + 1) = 44 \checkmark$

4) 5, 1, -4, -5, -1, 4, 5 ✓
 $T(1) \checkmark$
 $-T(1) \checkmark$
 $-T(n) \checkmark$

b) 8, 3, -5, -8, -3, 5, 8 ✓

c) $x, y, (y-x), -x, -y, (x-y), x$



(b) (i) $2/8 \checkmark$
 (ii) $4/8 \checkmark$
 (iii) $0 \checkmark$
 (iv) $1/6 \checkmark$
 (v) $4/8 \checkmark$

6. $5 + 11(2) = 27 \checkmark$

(b) 192 ✓

(c) Starting with \$29 not \$5
 Each month \$2 more is put
 in than previous month, ∴
 take less time to accumulate
 next \$192. OR SIMILAR ✓

(d) 69 months ✓
 68.?? 1 mark.

7(a) (i) $\frac{10}{50} \checkmark$ (ii) $\frac{85}{200} \checkmark$

(b)	FT	PT	total	decimals no marks
P	102	78	180	
F	48	12 ✓	60 ✓	
Total	150 ✓	90	240	

Other values in table ✓

(c) 2nd table figures are
 1st table figures $\times 240/200$
 i.e. all % are the same ✓

8. e.g. flood, drought, bananas
 viewed as bad/good for health ✓

(b) Up then down ✓
 (ii) slight increase ✓

(c) $\frac{2.90}{2} + 4.00 + 3.50 + 2.80 + 3.20 \checkmark$
 $= 3.3375 \checkmark$ [-1 if not 4dp]

(ii) $0.1625 \checkmark$
 (iii) $0.1625 \checkmark$

(d) (i) on graph ✓

(ii) winter, spring ✓
 higher price than other seasons ✓

(iii) $t = 16 \checkmark$
 $CHA = 3.18 + (0.0406)16 \checkmark$
 $= 3.83 \checkmark$

Predicted value
 $3.8296 + 0.1625 = \$3.99 \checkmark$
 ✓(2)

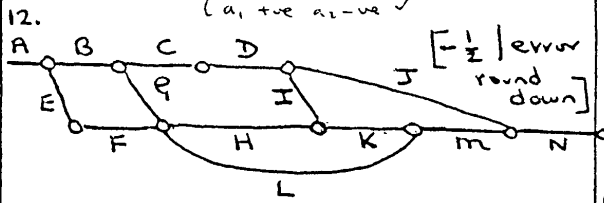
9a) i) 63.28125, 47.46 ✓
 (ii) on graph ✓
 (iii) on graph ✓ 24 ✓
 b) $A(t) = 200b^t$
 $150 = 200b^{10}$
 $A(t) = 200(0.9716)^t$
 (c) $100 = 200(0.9716)^t$ ✓
 $t = 24.094$
 $= 24 \text{ hrs. } 6 \text{ mins } \checkmark$

23 or 25 ok if shown on graph
 OR $t/10$
 $200(0.75)^t$
 $200(0.75)^t$
 -1 mark
 $-0.0287t$
 200e
 Linear regression (b)
 5 hrs
 28 hrs 12 min.
 FT.

10. \$47.29 ✓ $\times .30$ ✓
 \$7153.32 ✓
 \$7153.32 ✓
 Total interest = \$1328.78 ✓
 (b) $(20000 \times 0.1 \times 2)$ ✓ = \$4000 ✓
 (c) \$2671.22 ✓ $\times .30$ ✓ FT
 (d) No ✓ 10% interest always on full amount, 7% on what is owing ✓

ok
 47.24
 7153.32
 $\$1328.76$
 1328.81
 $\$1328$ or
 $\$1329$ ok

11. 15, 31 ✓
 (b) $2^n - 1$ ✓
 (c) $a_1 = a_2$ ✓ $a_1 = 5, a_2 = 5$ ✓
 (d) $a_1 > a_2$ ✓ $\{ a_1 = 2, a_2 = 1 \}$ ✓
 $\{ a_1 \text{ true } a_2 \text{ -ve } \}$ ✓



(b) ABCDJN ✓ 20 days ✓
 (c) Reduce ✓ to 18 days ✓
 reduce by 2 days ✓ ok
 13(a) True for 13 of 20 days in table } ✓
 From equation true for $x > 14$ } ✓
 (b) on graph ✓
 (c) 29.7 ✓

IF J max. cost with 1 for FT

14. (a) 19.757 m ✓
 (b) 25 m ✓
 (c) 8 ✓ or After 7th day ✓
 (i) 8cm or 80mm or 0.0806m ✓
 (d) $r = 10/\pi$ or $d = 20/\pi$ ✓
 Area = 31.83 m² ✓ $\times 31.85$
 (i) ✓ ✓ 3.45 31.724
 12.566 3.7165 ✓ 27.433
 ✓ 6.858 13.717 16.859
 (ii) 31.83 m² ✓ or circle

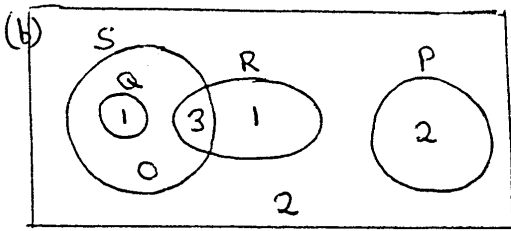
15. (8,8) ✓ (8,5) ✓ (5,5) ✓
 b) $y \geq 0$ ✓ $x \leq 8$ ✓
 $y \leq 5$ ✓ $x \geq y$ ✓
 c) Evaluate objective function at A, B & C ✓
 Compare answers ✓
 Take the maximum ✓

-1 if <
 0 = or none required

16. AEFQKL 39 [1/2 each round down]
 AEFQHKL 24
 AEFQHDCQKL 51
 AEFQHDCBFJKL 55
 ABFJKL 43
 ABFQHKL 55
 ABFQKL 40
 ABFQHDCQKL 72

(b) AEFQHDCBFJKL ✓
 (ii) ABFJKL ✓
 [1 mark for any other route of same length]

17(a) $\frac{5}{24}$ ✓ (-21) (-2083)
 0 ✓
 $\frac{3}{10}$ ✓
 $\frac{14}{24}$ ✓
 $\frac{7}{24}$ ✓



1/2 for identifying each region
 1/2 for correct number in region
 round down
 (1 mark if use elements AND all correct)

18. 80% ✓ Linear 80 ✓
 64% ✓ x
 $(0.8)^n 100$ ✓ x
 (b) 10 ✓ 4 ✓
 (c) $(0.9)^n 100$ ✓ x
 21 ✓ 9 ✓

(d) Cost/wash
 Bob: $6.99/10 = 0.699$
 Roberta: $14.50/21 = 0.690$ } ✓
 Roberta

19. \$552 000 ✓
 (b) 1.80 ✓✓
 [nearest 5c: \$1.75 ✓]
 [no rounding: \$1.76 ✓]
 (c) $\frac{10}{11} [345000 - 4(4000)] 1.80$

= \$538 363.64 ✓
 If we use \$1.75 ⇒ 529 772.73 FT

(d) 546 590.91 ✓
 561 954.55 ✓
 582 272.73 ✓

(e) 26 635 363.64
 27 635 590.91 * } ✓✓
 28 635 454.55

Price $\$1.60 + 27(0.05) = \2.95

$x = 27$ (1 mark)

$\$1.60 + 27.??(0.05) =$
 (2 marks)

$$20. (a) \quad 35 \checkmark \quad 51 \checkmark$$

$$(b) \quad 6 \checkmark \checkmark$$

$$(c) \quad (n+1)^2 \checkmark$$

$$(n+1)^2 + \frac{n(n+1)}{2} \checkmark \checkmark \quad \left[(n+1) \left(\frac{3n}{2} + 1 \right) \right]$$

$$\left[1.5n^2 + 2.5n + 1 \right]$$

$$21. (a) (i) \quad n+1 \quad \checkmark$$

$$(ii) \quad 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 \checkmark$$

$$(b) \quad (n+1)(m+1) \quad \checkmark \checkmark$$

$$(ii) \quad 500 = 2^2 \times 5^3 \quad \checkmark$$

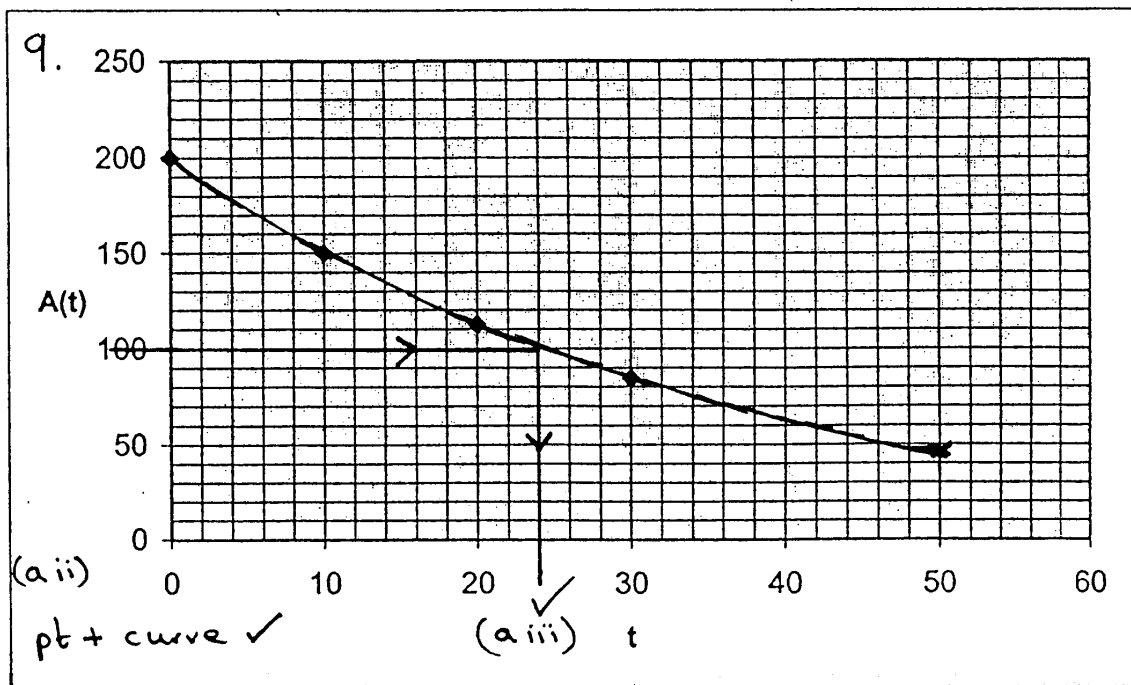
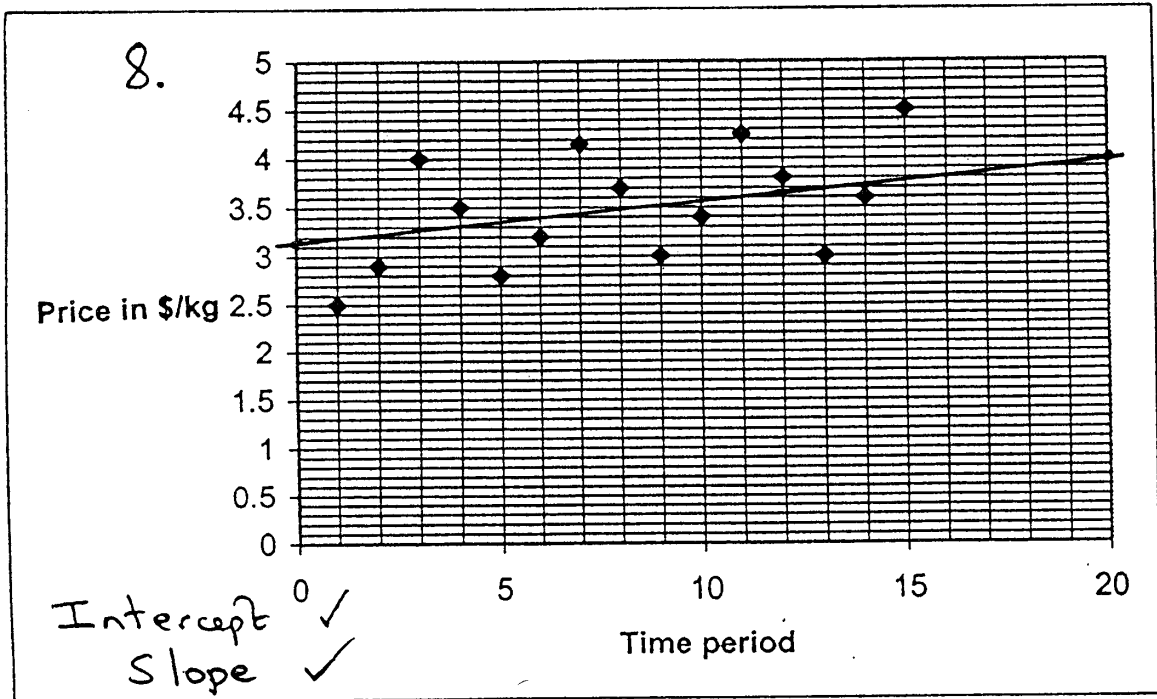
$$\text{number factors} = 3 \times 4 = 12 \quad \checkmark$$

Listed OK.

$$(c) (i) \quad (c_1+1)(c_2+1) \dots (c_k+1) \quad \checkmark \checkmark$$

$$(ii) \quad 9030 = 2 \times 3 \times 5 \times 7 \times 43 \quad \checkmark$$

$$\text{number factors} = 2 \times 2 \times 2 \times 2 \times 2 = 32 \quad \checkmark$$



13.

